A MODEL OF THE HIERARCHY OF NEEDS DYNAMICS

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A formal model is proposed for the hierarchy of needs' dynamics, where the degree of satisfaction of each individual need depends on the resource and the degree of satisfaction of the lower-level needs. Direct and inverse problems of resource allocation are solved, and the set of critical resources is determined. The conditions for attainability of the given level of need satisfaction are established, and the problem of resource allocation is reduced to the optimal control problem. *Copyright © 2007 IFAC*

Keywords: Agents, Human-centered Design, Hierarchies, Management Systems

1. INTRODUCTION

The concept of personal motivation which is now accepted by psychology and management, see (Mescom, *et al*., 1988), states that there exists a hierarchy of needs. A modification of the A. Maslow pyramid, see (Maslow, 1987), including seven ordered needs, such as physiological needs, physiological-and-psychological needs, psychological-and-social needs, social needs, interests of the functioning personality, interests of the developing personality, and strive for the cultural-and-creative activity was proposed by Verkhoglazenko, *et al*. (1998). The needs of the lower levels are called the primary needs, the rest of them are called the secondary needs. It is assumed that the individuals proceed to satisfying the needs of higher levels when those of lower levels are more or less satisfied. The present paper considers a formal model of the need hierarchy describing this qualitative effect. In this model, the degree of satisfaction of each need depends on the dynamics of allocated resource and the degree of satisfaction of the lower-level needs.

2. THE MODEL

Let there exist n ordered needs of which k first needs are primary.

The degree of satisfaction of the i-th need will be measured by the number $x_i \in [0; 1]$, $i \in N = \{1, ..., n\}$, where N is the set of needs.

We assume that the degree (level) of satisfaction of the i-th need depends on the resource (since the need is characterized as the want of something, it may be conventionally assumed that this «something» is a resource) $u_i \geq 0$ allocated for satisfaction of this need and on the degrees of satisfaction of lower-level needs:

$$x_i(u_1, ..., u_i) = \min \{f_i(u_i), \underline{\min}_{j=1,i-1} \alpha_{ij} x_j\},\ i \in N, \quad (1)$$

where f_i: $\Re^1_+ \to [0; 1]$ are certain strictly monotonic continuous functions and $\alpha_{ij} \in (0; 1]$ are the constants (weights), reflecting the interrelation between the needs, $j \leq i$, $i \in N$. In descriptive terms, these functions and constants reflect the individual characteristics of a human being, whose needs are modeled. Since actually any individual specifics can be taken into account by selecting the corresponding functions $f_i(\cdot)$ and constants $\{\alpha_{ij}\}$, the degree of satisfaction of the highest need

$$s(u) = x_n(u), \quad (2)$$

where $u = (u_1, ..., u_n) \in \Re_+^n$ is the resource vector, can be used as the aggregated degree of satisfaction of the needs $s \in [0; 1]$. For example, the value of $s(u)$ can be interpreted as the degree of employee's satisfaction with the job (if the resources for satisfaction of the needs are provided mostly by the organization), or readiness to work in the given organization (when changing work, the employee compares the current value of (2) with the alternative value proposed at the new place), and so on.

We note that the following qualitative results of analysis of the formal model remain valid if the aggregated degree of need satisfaction will be a monotonic continuous function of the degrees of satisfaction of the individual needs. For example, the aggregated degree of need satisfaction may be defined as a linear combination (weighted sum) of the degrees of satisfaction of individual needs. Then, the personal characteristics of an individual can be reflected by varying the weights.

If the functions $f_j(\cdot)$ achieve unit values for finite values of resources, then, given the value $x_1^{\max}$ of the maximal possible degree of need satisfaction of the lower level, one can calculate the maximum possible values of the need satisfaction degrees $x_i^{\max}$, $i \in N$. For that, we consider the graph (N, E), where the set of arcs E is the collection of the arcs from each vertex (corresponding to a need) to all need-vertices of a higher level. Let us calculate the «potential» of the i-th vertex

$$x_i^{\max} = \min_{j<i} (x_j^{\max} \alpha_{ij}), \; i \in N \setminus \{1\}. \quad (3)$$

The need satisfaction degree can be determined from (1) and (2) for the given functions $f_i(\cdot)$ and the resource vector u.

It is possible to solve the inverse problem of determining the minimal resources $u^*(s^*)$, providing the desired level

$$s^* \leq x_n^{\max} \quad (4)$$

of need satisfaction.

We denote by $\boldsymbol{\alpha} = ||\alpha_{ij}||_{i,\, j \in N}$ the weight matrix (the weight α_{ii} is assumed to be unity, $i \in N$); $f_i^{-1}(\cdot)$ – the inverse function of $f_i(\cdot)$, $i \in N$; $l_{ij} = ln(1 / \alpha_{ij})$; L_i – the length of the maximal path on the graph (N, E) from the vertex i to the vertex n, provided that the arc lengths are equal to l_{ij}, $i \in N$.

If for finite values of the resource the functions $f_j(\cdot)$ assume values s^*, then one can easily check, that the solution of the inverse problem is as follows:

$$u_i^*(s^*, \boldsymbol{\alpha}) = f_i^{-1}(s^* \exp (L_i)), \; i \in N. \quad (5)$$

<u>Statement 1.</u> The minimal values of the resources, enabling attainment of the given level $s^* \leq x_n^{\max}$ of need satisfaction, obey expression (5).

The resource allocation principle (5) can be called «uniform». Together with (1), it reflects the hierarchical structure of the personal needs. Now, we extend this model to the dynamic case. We recall that until now we disregarded the difference between the primary and secondary needs.

3. DYNAMICS

Let it be possible to spend Q units of the total resource in unit time; this total resource is time-constant. We denote by q_i the amount of resource, allocated in time unit for satisfaction of the i-th, $i \in N$, need. We assume for simplicity that this amount is time-constant.

We assume that the primary needs are not saturable (for example, any human being has to eat every day and his/her need in meal cannot be saturated), that is, $u_i = q_i$, $i = \overline{1,k}$, and the secondary ones are satisfiable, that is, $u_i(t) = q_i\, t$, $i = \overline{k+1,n}$ (for example, such results of creative activity as novels, poems, symphonies, etc., being once created, live for centuries). This assumption reflects the fact that the physiological needs, for instance, must be satisfied at each time instant and that the results of this satisfaction cannot be «accumulated», whereas the results of satisfying the creative needs may live for centuries in the works of art, scientific theories, and so on.

We assume here and below for simplicity that $\alpha_{ij} = 1$, $i \in N$, $j \leq i$. Then, $L_i = 0$, $i \in N$, and we obtain the following equations of the dynamics of the degrees of need satisfaction vs. the vector $q = (q_1, ..., q_n)$ of resources consumed in unit time:

$$x_i(q_1, ..., q_i, t) = \min_{j=\overline{1,i}} f_j(q_j), \; i = \overline{1,k}, \quad (6)$$

$$x_i(q_1, ..., q_i, t) = \min \{ \min_{j=\overline{1,k}} f_j(q_j), \; \min_{m=\overline{k+1,i}} f_m(q_m\, t)\}, \quad i = \overline{k+1,n}. \quad (7)$$

We note that all results hold also for arbitrary α_{ij}, in the corresponding expressions one only has to take into account the constants $\{L_i\}_{i \in N}$ – see (5).

The resource vector must satisfy the balance constraint

$$\sum_{i \in N} q_i \leq Q. \quad (8)$$

We obtain the condition for attainability of the level of need satisfaction s^* in finite time.

<u>Statement 2.</u> To attain in finite time the aggregated level of need satisfaction $s^* \leq x_n^{\max}$, it suffices to meet the condition

$$\sum_{i=1}^{k} f_i^{-1}(s^*) < Q. \qquad (9)$$

If condition (9) is satisfied, then by assuming that $q_i = f_i^{-1}(s^*)$, $i = \overline{1, k}$, we obtain from (6) that

$$x_i(q_1, ..., q_i, t) = s^*, \; i = \overline{1, k}. \qquad (10)$$

We fix $n - k$ strictly positive constants δ_i, $i = \overline{k+1, n}$, such that

$$\sum_{i=k+1}^{n} \delta_i = Q - \sum_{i=1}^{k} f_i^{-1}(s^*).$$

These constants exist by virtue of condition (9).

We denote $\delta = (\delta_{k+1}, ..., \delta_n)$ and assume that $q_i = \delta_i$, $i = \overline{k+1, n}$. At that, condition (8) is satisfied as equality.

From (7) and (10) we establish that the minimal time $T(\delta)$, after which the desired value s^* of the aggregated degree of need satisfaction is reached, is as follows:

$$T(\delta) = \max_{m=k+1,n} \{ f_m^{-1}(s^*) \, / \, q_m \}. \qquad (11)$$

This time is finite by virtue of (i) the strict monotonicity and continuity of the functions $f_i(\cdot)$ (see above) and (ii) the condition $s^* \leq x_n^{\max}$, which proves Statement 2.

We note that condition (9) implies that the resource available must be sufficient for satisfying the primary needs. Otherwise, the entire resource will be used by the nonsaturable primary needs, and nothing will be left to satisfy the secondary (saturable) needs.

4. THE MANAGEMENT PROBLEM

Let us consider now the problem of minimization of the time T of reaching the given level $s^* \in [0; 1]$ of satisfaction of the individual needs by means of resource allocation under the given resource constraints. The minimal time (the result of solving this problem) is denoted by T^*.

As follows from the proof of Statement 2 (condition (11), in particular), the following assertion is true. Its basic idea lies in that all secondary needs must reach the desired level simultaneously.

Statement 3. If $s^* \leq x_n^{\max}$ and condition (9) is satisfied, then the solution of the problem of speed is as follows:

$$q_i = f_i^{-1}(s^*), \; i = \overline{1, k},$$

$$q_m = \frac{f_m^{-1}(s^*)}{\sum_{l=k+1}^{n} f_l^{-1}(s^*)} \left(Q - \sum_{i=1}^{k} f_i^{-1}(s^*)\right), \; m = \overline{k+1, n},$$

$$T^*(s^*, Q) = \frac{\sum_{l=k+1}^{n} f_l^{-1}(s^*)}{Q - \sum_{i=1}^{k} f_i^{-1}(s^*)}.$$

The relations obtained also allow one to solve the problems of terminal control – minimization of the total resources for reaching the desired degree of need satisfaction or maximization of the aggregated level of need satisfaction in the given time and under the fixed resource constraints.

5. CONCLUSION

Thus, a formal model of the hierarchy of needs, where the degree of need satisfaction depends on the resource and the degrees of satisfaction of the lower-level needs, was proposed. The problems of resource allocation were solved for the static model (Statement 1). For the dynamic model, the conditions for attainability of the given level of need satisfaction (Statement 2) were obtained, and the problem of resource allocation to the needs of an individual was reduced to that of optimal control (Statement 3).

Consideration was given above to the problem of resource allocation for satisfaction of the needs of an individual person. The results obtained enable one to pose and solve the problems of resource allocation for motivation of several employees. Some possible approaches, as well as the difficulties of model identification, are discussed in (Novikov and Ivashenko, 2006).

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