Systems Science. 1999. Vol. 25б N 2. P.5-14.

Keywords: project management, resource allocation, network planning

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MODELS AND METHODS OF MULTIPROJECTS' MANAGEMENT

The paper deals with the problems of resource allocation between several independent projects. The problem is to minimise the time of all the projects implementation or the weighted sum of termination times. The described approach is based on the representation of a project as the separate operation. Then the problem of optimal resource allocation between the operations is solved. Given resource allocation for every project, one can solve the problem of allocation inside the multiproject. Allocation and aggregation methods are explored.

INTRODUCTION

The development of the society, economy, enterprise and the development of the certain man may be represented as the set of discrete processes with given terminal goals under the constraints on time and resources. It is convenient for a man to divide the process of his activity on local processes. Projects are processes of changes, i.e. nonrepeated processes, which require for their implementation special methods of management. For example, regular morning exercising is not a project as it is repeatable and does not require special organisational efforts. But learning new morning exercises may be considered as a project. Daily production on the enterprise is not the project too, but introduction of new technology is a project. Evidently, the difference is not obvious - construction of the building is a project, but there is no reason to consider the standard block lodges production with their installation on certain place as a project.

In the former Soviet Union project management was broadly used since the end of 60-th and was referred to as the network planning and management.

The base of project management is the presentation of the project network graphic, which reflects the dependence between different operations of the project. In 70-th the interest to the methods of network planning and management decreased, as the reasons of nonefficient management were deeper - they laid in the basis of the public-political and economic principles of state. Nowadays in Russia project management outlives a second birth. The Russian Association of Project Management (SOVNET) is the member of the International Project Management Association (INTERNET).

Multiprojects are an important class of projects. *Multiproject* is a project, which consists of several technologically independent projects, united by the shared resources (financial or material, etc.). This paper considers methods and mechanisms of multuprojects management. The described approach is based on the presentation of a project as the separate operation. Then the problem of optimal resource allocation between the operations is solved. Given resource allocation on every project, one can solve the problem of allocation for the multiproject (between the projects - operations of the multiproject). Several allocation and aggregation methods are explored below.

1. RESOURCE ALLOCATION BETWEEN INDEPENDENT OPERATIONS

Consider the multiproject, which consists of n independent projects. Each project is aggregately described as the operation with two characteristics: the volume of the project - W_i and the dependence between the rate of project implementation $w_i(t) = f_i(u_i(t))$ and the amount of resources $u_i(t)$ at time t. The volume, the velocity and the termination time T_i are interconnected trough the following equation:

$$\int_0^{T_i} f_i\left[u_i(t)\right]dt = W_i \,. \tag{1}$$

Let the total amount of resources for the multiproject is given and equals N(t). The problem is to allocate this resources between the operations so, that the termination time $T = \max_i T_i$ be minimal. If for any t N(t)=N (even arrival of resources) and $f_i(u_i)$ are concave functions, the resource allocation problem has the solution, defined in [1, 2, 3]. It is well known that the optimal allocation is characterised by the following properties:

à) each operation is implemented under the fixed level of resources $u_i(t)=u_i$, $i = 1 \div n$, $t \in [0, T]$, i.e. with a fixed rate;

á) all the operations terminate simultaneously.

Thus, $w_i = W_i/T$ is a constant rate of i-th operation. Denote $\varphi_i(w_i)$ - the function, inversed to function $f_i(u_i)$, then $u_i = \varphi_i(W_i/T)$ is the amount of resources to terminate i-th operation at time T. Minimal value of T is defined by the following equation

$$\sum_{i=1}^{n} j_i \left(\frac{W_i}{T} \right) = N . \qquad (2)$$

Let N(t) be piece wise constant function: $N(t) = N_k$, $t \in [\tilde{T}_{k-1}, \tilde{T}_k)$, $k = \overline{1, p}$, $\tilde{T}_0 = 0$. Fix some k and consider the problem of multiproject's implementation at the time, less than $\tilde{T}_k$. Denote x_{iq} - the volume of i-th operation, implemented in the q-th interval. Obviously

$$\sum_{q=1}^{k} x_{iq} = W_i . \qquad (3)$$

As in the interval $[T_{q-1}, T_q)$ the resources arrive evenly, the values $\{x_{iq}\}$ in the optimal solution satisfy the following constraint:

$$\sum_{i=1}^{n} \varphi_i \left(\frac{x_{iq}}{T_q} \right) = N_q , \quad q = \overline{1, k-1} , \quad T_q = \tilde{T}_q - \tilde{T}_{q-1}$$

and for the last interval the following condition holds:

$$\sum_{i=1}^{n}\varphi_i\left(\frac{x_{ik}}{T}\right)=N_k,\; T=\tilde{T}-\tilde{T}_{k-1},$$

where T is the time of all the operations termination.

Denote h_q - the vector with components $h_{iq} = \varphi_i'(w_{iq})$, $i=\overline{1,n}$. The vector h is «time-invariant» (its direction does not alter with the change of time interval, while its length changes):

$$h_q=\gamma_q h_1,\quad q=\overline{2,k}\;,\;\; h_1=\{h_i\},\; \gamma_1=1.$$

This characteristic of the optimal solution allows to reduce the resource allocation problem to the solution of the non-linear equations system with (n + k) variables $\{h_i\}$, $i=\overline{1,n}$, $\{\gamma_q\}$, and T:

$$\sum_{i=1}^{n}\varphi_i\left[\xi_i\left(\gamma_q\cdot h_i\right)\right]=N_q\;,\;\; q=\overline{1,k} \tag{4}$$

$$\sum_{q=1}^{k-1}\xi_i\left(\gamma_q\cdot h_i\right)\cdot T_q+\xi_i\left(\gamma_k\cdot h_i\right)\cdot T=W_i\;,\;\; i=\overline{1,n}. \tag{5}$$

Let $f_i(u_i)$ are concave functions, then, given the total amount of resources (financing) for the interval [0, T], maximum of the implemented operations volume is reached under even (homogenous) resources arrivement. The proved fact allows to optimise the schedule of resources arrivement. Such a tuning may be achieved by the shift of financing on later periods.

Hitherto we considered the problem of multiproject time minimisation. However, the other problem is not of the less important. The matter is that after the termination of any project one should receive some income. The delay in the termination leads to the decrease of income. Assume that i-th project returns after its termination the income c_i per time unit. Then the possible decrease of the income (lost income) is $c_i t_i$, while the total decrease equals

$$C=\sum_{i=1}^{n}c_i t_i\;. \tag{6}$$

Thus the following problem arises: to allocate resources between the projects to minimise (6).

2. THE GENERAL CASE

Above we have considered the problem of resource allocation under the assumption that the rate of project implementation is a concave function of the resources amount. Now we turn to the general case.

Let $f_i(u_i)$ be some limited, continuos on the right functions, such that $f_i(0)=0$. Define the set Y of pairs (u, w) in the following way:

$$Y_i = \{(u_i, w_i) > 0: \quad w_i \le f_i(u_i)\}. \qquad (7)$$

Note, that if $f_i(u_i)$ is a concave function, then Y_i is a convex set. Generally Y_i is not a convex set. The convex shell of this set is the convex set $\tilde{Y}_i$, such that any point may be represented as the convex linear combination of the points from the set Y_i. The border $\tilde{f}_i(u_i)$ of this set is, obviously, a concave function. Consider the resource allocation problem for the multiproject with the following rates of projects implementation { $\tilde{f}_i(u_i)$ }. Suppose, that we have some optimal allocation $\{u_i^0\}$ and operations termination times { t_i }.

Theorem 1. There exists feasible resource allocation with the same operations termination times t_i.

3. AGGREGATION METHODS FOR A COMPLEX OF OPERATIONS

Consider the methods of the complex of operations description in the form of some unique operation. One should determine the volume of the aggregated operation Wý, which is referred to as the equivalent volume of the complex, and the dependence between the rate of the aggregated operation implementation and the amount of resources, allocated for the complex:

$$W_ý(t) = F_ý(N(t)) \qquad (8)$$

Definition. Aggregation is identified as ideal if for any function N(t) there exists optimal resource allocation $\{u_i(t)\}$ between the operations of the complex, such that

$$\sum_{i=1}^{n} u_i(t) \le N(t),$$

meanwhile minimal termination time of the complex satisfies the following condition:

$$\int_0^{T_{min}} F_э(N(t))dt = W_э.$$

The classical example of the ideal aggregation is the following :

$$f_i(u_i) = u_i^{\alpha}, \quad \alpha \le 1, \quad i = \overline{1, n}. \qquad (9)$$

Really, for this case it was proved [3], that there exists equivalent volume Wý of the complex and the function

$$F_ý(N) = N^{\alpha},$$

such that for any resources level N(t) the following condition holds

$$\int_0^{T_{min}} F[N(t)]dt = W_э.$$

Herewith, there exists optimal allocation $\{u_i^0(t)\}$, such that

$$\sum_{i=1}^{n} u_i^0(t) = N(t),$$

and complex's termination time equals T_{min}.

Let for any t N(t) = N, then the allocation $\{u_i^0(t)\}$ possesses the following interesting characteristics:

1. Each operation is implemented without any breaks with the constant amount of resources, i.e.

$$u_i^0(t) = u_i, \quad t \in \left[t_i^H, t_i^0\right],$$

where t_i^H, t_i^0 are moments of the i-th operation beginning and termination.

2. The resources $\{u_i\}$ form the flow on the network graph.

Let us describe the algorithm of the equivalent volume of the complex determination. First one should define the dependence between costs $S_{ij} = u_{ij} \cdot \tau_{ij}$ and operation's time:

$$S_{ij} = \tau_{ij} \cdot \left(\frac{w_{ij}}{\tau_{ij}} \right)^{1/\alpha} = \frac{w_{ij}^{1/\alpha}}{\tau_{ij}^{1-\alpha/\alpha}}$$

Consider the problem of optimising the complex by cost: to determine operations times to implement the complex in time Ò with minimal costs

$$S = \sum_{(i,j)} S_{ij}\left(\tau_{ij}\right). \tag{10}$$

It is well known that necessary and sufficient conditions for optimality is the following: for any event i on the network (excluding initial and terminal ones):

$$\sum_{j} \frac{dS_{ij}\left(\tau_{ij}\right)}{d\tau_{ij}} = \sum_{k} \frac{dS_{ki}\left(\tau_{ki}\right)}{d\tau_{ki}}. \tag{11}$$

But, as

$$\frac{dS_{ij}\left(\tau_{ij}\right)}{d\tau_{ij}} = -\frac{1-\alpha}{\alpha} u_{ij},$$

the condition (11) is equivalent to the flowence condition for the resources at any event. The minimum of the costs, under the condition that resources form the flow in the network, is equivalent to the minimum of the resources amount N.

To minimise the costs on the network the efficient Kelly algorithm may be applied. On each step of this algorithm condition (11) is tested for the nodes of the network, and the time of the corresponding event is corrected to satisfy this conditions, thus the value of (10) decreases. When some solution satisfies (11) for all the nodes, the value of costs (10) is minimal and, consequently, total amount of resources N_{min} is minimal too. Equivalent volume of the complex is defined by

$$W_ý = TN^{\alpha} = S^{\alpha}_{min} \cdot T^{1-\alpha}$$

Let us describe another method of the equivalent complex's volume estimation. This method is based on the following theorem:

Theorem 2. Equivalent volume of the complex is a convex homogenous function of operations volumes $\overline{W}$.

The theorem allows to obtain overestimates of the equivalent volume of the complex, due to the presentation of the complex in the form of the convex linear combination of other complexes with known equivalent volumes.

Example. Consider the complex of operations, presented of fig. 1. The precise value of complex's equivalent volume equals $W_ý = 3\sqrt{41} \approx 19{,}2$.

Consider two complexes of operations, presented on fig. 2 à, b. It easy to see that the average of this complexes volumes gives the first complex. For the second complex we obtain

$$W^1_э = \sqrt{21^2 + 13^2} \approx 24{,}7,$$

and for the complex from fig. 2.b:

$$W^2_э = \sqrt{11^2 + 8^2} + 5 \approx 18{,}6.$$

Thus for the original complex:

$$W_э \le \frac{1}{2}\left(W^1_э + W^2_э\right) = 21{,}6.$$

The deviation from the precise estimate is 2,4 or approximately 12,5%.

Accuracy of the estimate may be improved by the selection of different values W^1, W^2 and α, such that

$$W = \alpha W^1 + (1 - \alpha)W^2.$$

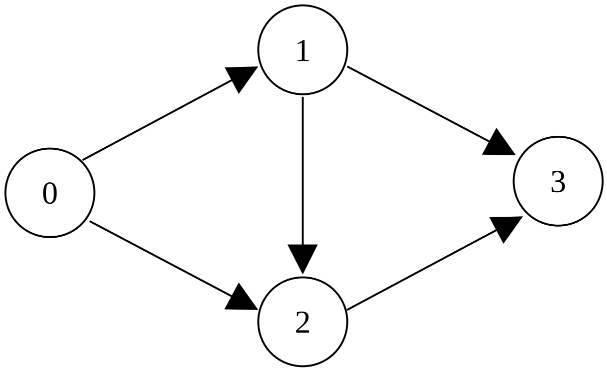

Fig. 1.

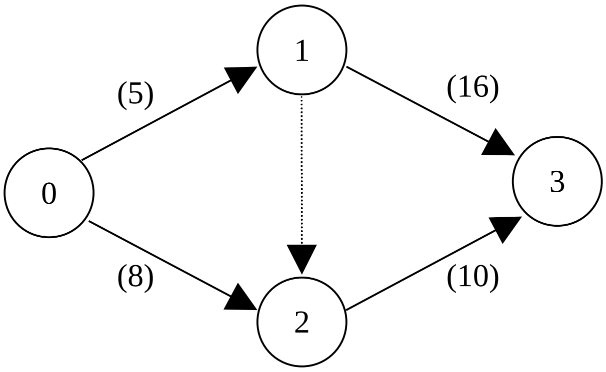

Fig. 2.à

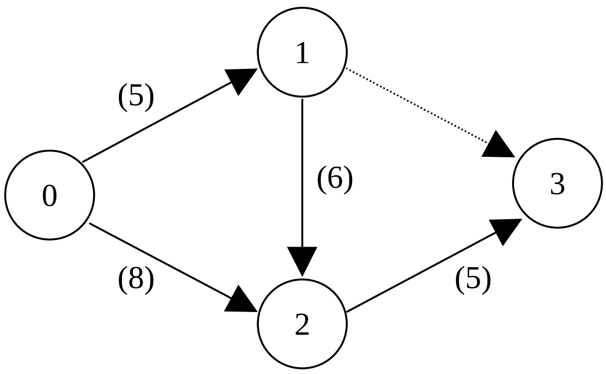

Fig. 2.b

For example, if $\alpha \approx 1$, then:

$$W_ý^1 \approx 18, \;\; W_ý^2 \approx 3/(1-\alpha), \;\;\; W_ý \leq 18 + 3 = 21,$$

i.e. the deviation is approximately 9%.

4. THE LINEAR CASE

Consider the linear dependence between the rates of operations and the amount of resources:

$$f_i(u_i) = \begin{cases} u_i, & u_i \leq a_i \\ a_i, & u_i \geq a_i \end{cases}.$$

Denote $\tau_i = w_i/u_i$ - the minimal time of i-th operation. Let us construct the integral graphic of resource utilisation on the complex of operations under the assumption that all the operations are initialised at the latest time moments. Graphic of resources utilisation is an aggregated description of the project. Really, given the aggregated descriptions of all the projects (right-shifted graphics of the projects), we are able to solve the problem of the optimal resource allocation for the multiproject both for the criteria of time minimisation and for the lost income minimisation.

Time minimisation problem for the multiproject is solved rather simple. Let $S_i(t,T)$ be the integral graphic of i-th project resources utilisation under the condition of its termination at time moment T, $S(t,T) = \sum_{i=1}^{n} S_i(t,T)$ - the integral graphic of multiproject resources utilisation, M(t) - the integral graphic of total amount of resources (fig.. 3).

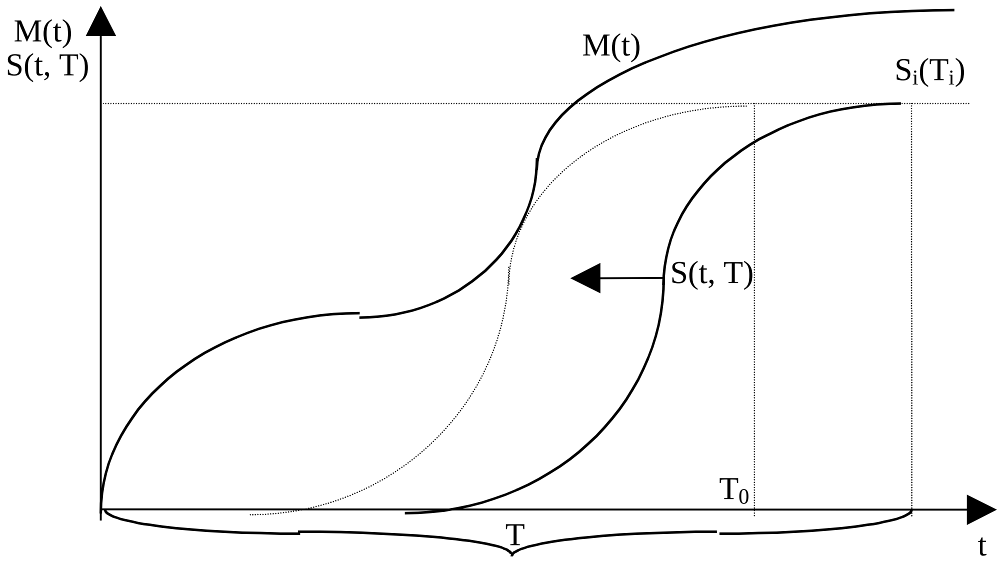

Fig. 3.

To determine the minimal time T_0 of the multiproject implementation one should shift the graphic S(t, T) to the left (see fig. 3) until it will touch the graphic M(t). Having got integral feasible graphic of resource allocation for the multiproject, one can split it into the graphics of resource allocation for certain projects (the process of decomposition). The lost income minimisation problem is not charactarized by such an «easy» solution method. If the number of projects is not very high, it may be solved by the analysis of all the possible combinations. It's worth noting that if the priority of the projects is fixed, then the allocation is obtained by the shift (in given sequence) of the integral graphics $S_i(t, T)$ to the left until touching the graphic M(t). If the number of the projects is high enough, heuristic rule, given in section 2, may be applied.

CONCLUSION

The paper contains the development of the optimal resource allocation methods for multiprojects.

The idea of the decomposition, based on aggregation of the projects and the consequent solution of resource allocation problems for the set of independent operations, turned to be extremely fruitful. It is important that for most of the practically used in the project management dependencies of operations rates from the amount of resources (linear, exponential, etc.), the ideal aggregation is possible and adequate.

Moreover, if the parameters of the projects are not common knowledge, then the game-theoretical problems of truthtelling and coordinated planning arise. Some approaches to the construction of project management mechanisms, which are efficient, coordinated and non-manipulable, are described in [4,5].

The aggregation problem for the projects with much more complex dependencies of operations rates from the amount of resources requires further studies. The solution of the applied problems of projects financing in the framework of the developed approach, also seems rather perspective.

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