TWO-LEVEL ACTIVE SYSTEMS.

III. EQUILIBRIA IN ABOVE-BOARD CONTROL LAWS

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This paper is concerned with functioning mechanisms of two-level active systems in which the control laws rely on the principle of above-board control [1]. Several properties of the solutions to a game of active elements [2] in active systems with such functioning mechanisms are proved. As the solutions of a game of active elements we consider Nash equilibrium situations [3] under the hypothesis of weak influence.

1. Introduction

Of the present series of papers, [1] dealt with the description of a model of a two-level system and its functioning mechanisms, and [2] posed the problems of analysis and design of functioning mechanisms with the counterflow method of data formation. In the approach developed in [2] the analysis and design of functioning mechanisms of active systems reduces to the investigation of a special class of games with nonconflicting interests [4] of the center (C) and the active elements (AE). At present, there are still no regular methods developed for the solution of such problems. One projected way of solving the design problem consists of choosing "good" (from economic, practical, and other considerations) functioning mechanisms and subsequently verifying their properties. In this scheme the functioning mechanisms that realize in various forms the idea, put forth by Soviet economists, of coordination of the interests of the system on the whole and of the elements that compose it seem promising [5]. A version of realization for the idea of coordination of interests in an organizational system is application of control laws based on the principle of coordinated control, and, in particular, the principle of above-board control [1]. This paper deals with the equilibrium situation in functioning mechanisms that use the laws of above-board control.

2. Relevant Solutions of a Game of Active Elements

In our investigation of a game with C and AE, solutions of the game of the AE will be understood to be Nash equilibrium situations [3] under the additional assumption that each AE does not take into account the influence of the estimate s_i communicated by it to the control $\lambda(s)$ established by the center (the hypothesis of weak influence as a hypothesis about the behavior of the elements, or, briefly, the WI hypothesis).

For the functioning of an active system (AS) it is possible to give several variants such that the solutions of the game of the active elements should be taken to be Nash equilibrium situations. In several AS models with repeating periods of functioning the indicator behavior of the AE leads to such solutions of the game (Example 4 in [2]). A number of collective principles of choice of rational strategies can also be reduced to a Nash equilibrium situation (Example 3 in [2]). We can also mention the case when there exist "absolutely optimal" strategies for the elements (Example 2 in [2]). The solutions of the game that are obtained for a choice of "absolutely optimal" strategies by the elements satisfy the Nash equilibrium conditions.

The following circumstances form the basis for the assumption by the center of the hypothesis of weak influence. For a quite large number of enterprises and in the absence of a monopolistic effect in the branch, the facts of weak influence of a particular enterprise on the control parameters that are common for all the enterprises (prices, norms) and that are determined from an analysis of the state of all the enterprises of the branch are well known in real organizational systems in economics. It is also possible to draw a certain analogy with the condition of "perfect competition" in models of a decentralized economy [6] and others, which assumes that the elements (producers and consumers) do not take into account the influence of the significance of the characteristic realization on the price vector. Undoubtedly, the assumption of the WI hypothesis is justified if in a certain sense there really is a weak influence of an estimate s_i on the control $\lambda(s)$. We shall assume that the WI hypothesis is plausible for a sufficiently large number of elements if the following condition of weak influence is satisfied (we assume that $\lambda(s)$ are continuously differentiable functions and $\forall s \in \Omega, \forall j: \lambda_j(s) \geq \lambda_{j\,min} > 0$):

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$$\forall s \in \Omega,\ \forall i, j, l: \lim_{n\to\infty} \frac{\partial \lambda_j(s)}{\partial s_{il}} = 0. \tag{1}$$

Here n is the number of AE in the system. Sufficient conditions for the satisfaction of the condition of weak influence (1) for resource allocation models were given in [7-9]. Further, we assume that the condition of weak influence holds, and, consequently, it is possible to assume the WI hypothesis.

We write the conditions of Nash equilibrium for the n AE with the effectivity criteria (1) in [2].

The situation $s^* = \{s_i^*\} \in \Omega$ is called a Nash equilibrium if

$$\forall i: \varphi_i(\lambda(s^*), x_i(s^*), r_i) = \max_{s_i \in \Omega_i} \varphi_i(\lambda(s^*(i)), x_i(s^*(i)), r_i), \tag{2}$$

where $s^*(i) = (s_1^*, \ldots, s^*_{i-1}, s^*_{i+1}, \ldots, s_n^*)$. Under the assumption of the WI hypothesis the Nash equilibrium situations s^* are determined by the conditions

$$\forall i: \varphi_i(\lambda^*, x_i(s^*), r_i) = \max_{s_i \in \Omega_i} \varphi_i(\lambda^*, x_i(s^*(i)), r_i), \tag{3}$$

where $\lambda^* = \lambda(s)$.

For the subsequent presentation we need the following lemma (the results of this and the following section are given under the assumption that all the components of the realization vectors of the AE are planned; a generalization to the case of functioning mechanisms with partial planning of realizations of elements is given in the appendix).

Lemma 1.

$$\forall \lambda \in L,\ \forall i: \max_{x_i \in R_+^{p_i}} \varphi_i(\lambda, x_i, r_i) = \max_{x_i \in X_i(r_i)} \varphi_i(\lambda, x_i, r_i) = \max_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i); \tag{4}$$

$$\forall \lambda \in L,\ \forall i: \operatorname{Arg}\max_{x_i \in R_+^{p_i}} \varphi_i(\lambda, x_i, r_i) = \operatorname{Arg}\max_{x_i \in X_i(r_i)} \varphi_i(\lambda, x_i, r_i) = \operatorname{Arg}\max_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i). \tag{5}$$

Here p_i is the dimension of the vector x_i, $R_+^{p_i}$ is the positive orthant of p_i-dimensional Euclidean space, and the symbol "$\operatorname{Arg}\max_{z \in Z} f(z)$" denotes the set of all z^* such that $f(z^*) = \max_{z \in Z} f(z)$.

A qualitative interpretation of this result is as follows. The maximum of the efficiency criterion of each element with respect to the plan is attained on the set of realizable plans; moreover, because of the presence in the goal functions of the elements of penalties for deviation of the realization from the plan, the optimal plan for an element coincides with the realization ensuring a maximum for its goal function on the set of locally admissible realizations for zero penalties.

Corollary. For the functioning mechanism [1] $\Sigma = \langle W, B, \pi \rangle$ a sufficient condition for the existence of a Nash equilibrium (3) under the hypothesis of weak influence is the following: There is a point s^* at which the control $\lambda^* = \lambda(s^*)$ and the plan $x^* = x(s^*)$ satisfy the conditions

$$\forall i: \varphi_i(\lambda^*, x_i^*, r_i) = \max_{x_i \in X_i(r_i)} \varphi_i(\lambda^*, x_i, r_i). \tag{6}$$

Indeed, for any equilibrium situation s^* we can write the following chain of conditions:

$$\forall i: \varphi_i(\lambda^*, x^*, r_i) = \max_{s_i \in \Omega_i} \varphi_i(\lambda^*, x_i(s^*(i)), r_i) \leqslant \max_{x_i \in R_+^{p_i}} \varphi_i(\lambda^*, x_i, r_i) = \max_{x_i \in X_i(r_i)} \varphi_i(\lambda^*, x_i, r_i).$$

Here we have written first on the left the equilibrium condition (3), the next inequality is obvious, and the next equality is satisfied in view of Lemma 1. From this it is clear that if (6) holds for $s^* \in \Omega$, then s^* is an equilibrium situation.

3. Analysis and Design of Functioning Mechanisms in the Class G^1

We say that the functioning mechanism $\Sigma = \langle W, B, \pi \rangle$ belongs to the class G^1 if, under the assumption of the WI hypothesis, we have for it:

a) there exists at least one situation s^* of Nash equilibrium;

b) any situation s^* of Nash equilibrium satisfies (6).

Nash equilibrium situations under the WI hypothesis (6) (in the following we shall for brevity call them simply equilibrium situations) were first introduced in [10]. In the same place it was conjectured that in the class G^1 the functioning mechanisms $\Sigma = \langle W, B, \pi_{abc} \rangle$ that include the laws of above-board control guarantee the reliability of the information communicated by the elements and are an optimal solution to the problem of choosing a control law. A rigorous justification of this result was first carried out on examples of a series of one-dimensional models of AS [7, 10], then on a multidimensional model of resource allocation [8, 9]; in this paper it is carried out for the general model of an AS described in [1].

The following lemma proves that under the assumption of the WI hypothesis the functioning mechanisms $\Sigma = \langle W, B, \pi_{abc} \rangle$ belong to the class G^1.

Lemma 2. If in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ the WI hypothesis holds, then $\Sigma = \langle W, B, \pi_{abc} \rangle \in G^1$.

The fact that an equilibrium situation $s^* = r$ exists in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ is established by the next result.

Theorem 1. If in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ the WI hypothesis holds, then the situation $s^* = r$ is always an equilibrium in the sense of (6).

Corollary. If in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ the WI hypothesis holds and there exists only one equilibrium situation s^* (6), then it has the form $s^* = r$. Correspondingly, in this case the degree of distortion of information (13) in [2] is $\delta = 0$.

Remark 1. If in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ the WI hypothesis holds, then for each AE the equilibrium strategy $s^*_i = r_i$ is "absolutely optimal," $i \in I$ (proof in the Appendix).

We denote by $G^1(W, B)$ the set of all control laws π' such that under the fixed system of stimulation (W, B) any functioning mechanism $\Sigma = \langle W, B, \pi' \rangle$ belongs to G^1: $\Sigma = \langle W, B, \pi' \rangle \in G^1$. We consider the problem of choosing a control law [2] on the set $G^1(W, B)$. Theorem 2 solves this problem in favor of the laws of above-board control. The uniqueness of the equilibrium situation $s^* = r$ is an additional condition imposed here on $\Sigma = \langle W, B, \pi_{abc} \rangle$.

Theorem 2. If the WI hypothesis holds in an AS with functioning mechanism $\Sigma^* = \langle W, B, \pi_{abc} \rangle$ and there is a unique equilibrium situation $s^* = r$, then

$$K_{\Sigma^*} = \max_{\pi \in G^1(W,B)} K_{\langle W,B,\pi \rangle}. \tag{7}$$

The results of Theorem 2 show that under its assumptions the above-board control laws are optimal in the class $G^1(W, B)$. The value of the goal function of the AS in the equilibrium situation $s^* = r$ for these control laws is determined by the expression (A.11) (see the Appendix) and in the general case is not equal to its maximal possible value $\Psi_m(r)$ because of the conditions of coordination (see (11) in [2]). In this connection we can try to change the system of stimulation (W, B) in such a way as to increase the value of the effectivity criterion of the C or even to attain an absolutely optimal value of it [2], i.e., $K_{\Sigma^*} = 1$.

The next theorem states a condition on the functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ that is sufficient for its absolute optimality. First, following [10, 11], we introduce the condition of epicoordination of the functioning mechanisms $\Sigma = \langle W, B, \pi_{abc} \rangle$. We consider $\Psi_{rc} = \max_{\lambda, x} \Psi(\lambda, x, s)$,

where $\lambda \in L,\ x \in X(s)$ и $\Psi_{abc}(s) = \max_{\lambda, x} \Psi(\lambda, x, s)$,

where $\lambda \in L,\ x \in X(s),\ x_i \in \operatorname{Arg} \max_{x_i \in X_i(s_i)} \varphi_i(\lambda, x_i, s_i), \quad i \in I.$

These are the expressions for the optimal values of the goal function of an AS in the problems of rigid centralization and coordinated planning, respectively. We define the coordination coefficient as

$$\rho(s) = \Psi_{rc}(s)/\Psi_{abc}(s). \tag{8}$$

The functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ is said to be epicoordinated if for it the coordination coefficient $\forall s \in \Omega$ is $\rho(s) = 1$. Qualitatively, this condition means the "full coordination" of the goal of the whole AS and the goals of the AE that make it up.

Theorem 3. If in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ the epicoordination condition holds and there exists a unique solution $\hat{s} = r$ of the game of the AE, then $K_{\Sigma*} = 1$.

Corollary. If the WI hypothesis and the condition of epicoordination hold in an AS with functioning mechanism $\Sigma^* = \langle W, B, \pi_{abc} \rangle$, and there exists a unique equilibrium situation $s^* = r$, then $K_{\Sigma*} = 1$. Examples of epicoordinated functioning mechanisms $\Sigma = \langle W, B, \pi_{abc} \rangle$ can be found in [11].

In Theorems 2 and 3 essential use is made of the uniqueness of the equilibrium situation. In this connection, sufficient conditions for the existence of a unique equilibrium situation in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ are of great interest. The following theorem formulates such conditions in the form of conditions on the preference functions of the AE.†

We say that the condition of equality of rights of the preference functions for the i-th AE is satisfied if

$$\forall \lambda \in L,\ \forall s_i^1,\ s_i^2 \in \Omega_i,\ s_i^1 \neq s_i^2 : A_i(\lambda,\ s_i^1) \cap A_i(\lambda,\ s_i^2) = \phi, \tag{9}$$

where $A_i(\lambda,\ x_i) = \operatorname{Arg} \max_{x_i \in X_i(s_i)} \varphi_i(\lambda,\ x_i,\ s_i)$. In content this condition means that for any admissible distinct estimates s_i^1, $s_i^2 \in \Omega_i$, $s_i^1 \neq s_i^2$ of the i-th AE the corresponding sets of coordinated plans of the AE do not have intersections.

Theorem 4. If in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ the WI hypothesis and the condition of equality of rights of the preference functions for all the AE ($i \in I$) hold, then $s^* = r$ is the unique equilibrium situation.

Corollary. If in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ the WI hypothesis holds and $\forall \lambda \in L$, $\forall s_i \in \Omega_i$, the preference function of each AE has a unique maximum at the point $x_i = \varkappa_i(\lambda,\ s_i)$, where $\forall s_i^1,\ s_i^2 \in \Omega_i,\ s_i^1 \neq s_i^2 : \varkappa_i(\lambda,\ s_i^1) \neq \varkappa_i(\lambda,\ s_i^2)$, $i \in I$, then $s^* = r$ is the unique equilibrium situation. The validity of the assertion follows from the fact that the conditions formulated are sufficient for the satisfaction of the conditions of equality or rights of the preference functions of all the AE (9).

By this Corollary, the conditions of equality of rights of the preference functions of all the AE hold for the resource allocation problem, the "Plan" problem, and the "Consumer–Producer" problem [7-10].

Another sufficient condition for the existence of a unique equilibrium situation $s^* = r$ in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ is the introduction of penalties for distortion of information [1, 2]. In the course of the following presentation we assume that the operator $\sigma = \{\sigma_i\}$ of formation of the estimates $\theta = \{\theta_i\}$ where $\theta_i = \sigma_i(s_i,\ y_i)$, $i \in I$ [2], is such that $\forall i : \theta_i \neq s_i$, if $s_i \neq r_i$, i.e., if at the stage of data formation that AE communicate unreliable information, then at the realization stage of the plan the C, in generating the estimates $\theta = \{\sigma_i(s_i,\ y_i)\}$, can always determine this.

Theorem 5. If in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ the WI hypothesis holds and penalties for distortion of information are introduced in the goal functions of all the AE, then $s^* = r$ is the unique equilibrium situation.

It is interesting to note that in the condition of Theorem 5 no mention is made of the "force" of the penalties for distortion of information. It can be assumed that in practice the "effect of penalties" is always present in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ in the form of a moral stimulus to communicate reliable information $s^* = r$, the other conditions being equal; by "equal conditions" we mean here the fact that in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ for which the WI hypothesis holds each AE receives, for fixed equilibrium strategies of the remaining players $(s_1^*, \ldots, s^*_{i-1},\ s^*_{i+1}, \ldots, s_n^*)$, one and the same payoff, equal to $\max_{x_i \in X_i(r_i)} \varphi_i(\lambda(s^*), x_i, r_i)$, $i \in I$, in any equilibrium situation.

APPENDIX

Proof of Lemma 1. We use the notation

$$D_i(\lambda, r_i) = \operatorname{Arg} \max_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i) \subset Y_i(r_i). \tag{A.1}$$

†The approach to the proof of the theorem was formed in [12] in an investigation of the guaranteeing strategies of the elements in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$.

Let x_i' be some arbitrary plan of the i-th AE and y_i' any locally optimal realization of the i-th AE for the control λ and the plan x_i'. Then

$$f_i(\lambda, x_i', y_i') = \max_{y_i \in Y_i(r_i)} f_i(\lambda, x_i', y_i) = \varphi_i(\lambda, x_i', r_i). \qquad (A.2)$$

a) We prove that if $x_i' \notin D_i(\lambda, r_i)$ and $\forall\, y_i^* \in D_i(\lambda, r_i)$

$$f_i(\lambda, y_i^*, y_i^*) > f_i(\lambda, x_i', y_i') = \varphi_i(\lambda, x_i', r_i'). \qquad (A.3)$$

Two cases are possible: $y_i' = x_i'$ and $y_i' \neq x_i'$. Let $y_i' = x_i'$. Then (A.3) follows from the definition of the set $D_i(\lambda, r_i)$ (A.1). Let $y_i' \neq x_i'$. We assume that (A.3) does not hold, i.e.,

$$f_i(\lambda, y_i^*, y_i^*) = \max_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i) \leqslant f_i(\lambda, x_i', y_i') < f_i(\lambda, y_i', y_i'). \qquad (A.4)$$

The second inequality in this chain holds by the property (1) in [1] of the goal function of an AE with penalties for deviation of the realization from the plan. It is clear that (A.4) is a contradiction. This proves (A.3) for the case $y_i' \neq x_i'$.

b) We prove that if $x_i' \in D_i(\lambda, r_i)$, then

$$y_i' = x_i'. \qquad (A.5)$$

This follows from the next conditions:

$$f_i(\lambda, x_i', y_i') \begin{cases} = f_i(\lambda, x_i', x_i') = \max\limits_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i), & \text{if} \quad y_i' = x_i', \\ < f_i(\lambda, y_i', y_i') \leqslant \max\limits_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i), & \text{if} \quad y_i' \neq x_i'. \end{cases}$$

Thus, by (A.1), (A.3), and (A.5), we can write

$$\max_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i) \begin{cases} = \varphi_i(\lambda, x_i', r_i), & \text{if} \quad x_i' \in D_i(\lambda, r_i), \\ > \varphi_i(\lambda, x_i', r_i), & \text{if} \quad x_i' \notin D_i(\lambda, r_i), \end{cases} \qquad (A.6)$$

where $D_i(\lambda, r_i) \subset Y_i(r_i) = X_i(r_i) \subset R_+^{p_i}$. By the last condition, if $x_i' \in D_i(\lambda, r_i)$, then

$$\max_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i) = \max_{x_i \in X_i(r_i)} \varphi_i(\lambda, x_i, r_i) = \max_{x_i \in R_+^{p_i}} \varphi_i(\lambda, x_i, r_i) = \varphi_i(\lambda, x_i', r_i);$$

if $x_i' \notin D_i(\lambda, r_i)$, then

$$\max_{y_i \in Y_i(r_i)} f_i(\lambda, y_i, y_i) = \max_{x_i \in X_i(r_i)} \varphi_i(\lambda, x_i, r_i) = \max_{x_i \in R_+^{p_i}} \varphi_i(\lambda, x_i, r_i) > \varphi_i(\lambda, x_i', r_i).$$

The assertions (4.12) and (4.13) in [1] follow from this. The lemma is proved.

To prove Theorem 1 we need the following lemma.

Lemma 3. If in an AS with functioning mechanism $\Sigma \in G^1$ the WI hypothesis holds, then for each AE in any equilibrium situation s^* the realization y_i^* coincides with the plan $x_i(s^*)$:

$$y_i^* = x_i(s^*), \quad i \in I. \qquad (A.7)$$

Proof. For functioning mechanisms in G^1 any equilibrium situation s^* satisfies (6). Considering the conditions (4) and (5) given in the formulation of Lemma 1, we immediately get a proof of Lemma 3.

Proof of Theorem 1. We write the conditions for perfect coordination (A.6) in [1] for the law of aboveboard control (abc) when the estimates $s^* = r$ are communicated by the elements:

$$\forall i: \varphi_i(\lambda(s^*), x_i(s^*), r_i) \underset{s^* = r}{=} \max_{x_i \in X_i(r_i)} \varphi_i(\lambda(s^*), x_i, r_i). \qquad (A.8)$$

By (A.8), the equilibrium conditions (6) hold in the situation $s^* = r$. The theorem is proved.

Proof of Remark 1 to Theorem 1. It must be shown that the following condition holds:

$$\forall s \in \Omega, \ \forall i: \ \varphi_i(\lambda(s^1(i)), x_i(s^1(i)), r_i) = \max_{s_i \in \Omega_i} \varphi_i(\lambda(s), x_i(s), r_i). \qquad (A.9)$$

Here $s^1(i) = (s_1, \ldots, s_{i-1}, r_i, s_{i+1}, \ldots, s_n)$. We write the conditions for perfect coordination (A.6) in [1] for the law of abc and for the estimates $s^1(i)$ of the AE

$$\forall i:\ \varphi_i(\lambda(s^1(i)), x_i(s^1(i)), r_i) = \max_{x_i \in X_i(r_i)} \varphi_i(\lambda(s^1(i)), x_i, r_i) = \max_{s_i \in \Omega_i} \varphi_i(\lambda(s), x_i(s), r_i).$$

The second equality from the left are written in view of the Corollary to Lemma 1. With this, (A.9) is proved.

Proof of Lemma 2. Under the conditions formulated in the lemma, the existence of an equilibrium situation $s^* = r$ satisfying the conditions (6) follows from Theorem 2. It remains to prove that for any situation s^* of equilibrium (3) the conditions (6) hold.

We consider any $\forall i \in I$. If the equilibrium strategy of the i-th AE is $s_i^* = r_i$, then, by the conditions for perfect coordination for the abc law (A.6) in [1], the condition (6) holds for the i-th AE.

Let $s_i^* \neq r_i$. Two cases are possible: a) (6) holds; b) (6) does not hold. We prove by contradiction that under the conditions formulated in the lemma the case b cannot hold. Indeed, suppose that (6) does not hold, and, consequently, $x_i(s^*) \notin D_i(\lambda^*, r_i)$. We consider the following situation: $s^1(i) = (s_1^*, \ldots, s^*_{i-1}, r_i, s^*_{i+1}, \ldots, s_n^*)$. We prove that

$$\varphi_i(\lambda^*, x_i(s^1(i)), r_i) > \varphi_i(\lambda^*, x_i(s^*), r_i), \tag{A.10}$$

where $\lambda^* = \lambda(s^*)$. From this it follows that s^* is not an equilibrium situation, consequently, the case b cannot hold. For the proof of (A.10) we write the following chain of conditions:

$$\varphi_i(\lambda^*, x_i(s^1(i)), r_i) = \max_{x_i \in X_i(r_i)} \varphi_i(\lambda^*, x_i, r_i) > \varphi_i(\lambda^*, x_i(s^*), r_i).$$

The first equality in this chain holds by the definition of the conditions of perfect coordination (A.6) in [1], and the next strict inequality holds by Lemma 1, if we assume that $x_i(s^*) \notin D_i(\lambda, r_i)$. The lemma is proved.

Proof of Theorem 2. We use the notation $T_i(\lambda, x_i, r_i) = \operatorname{Arg} \max_{y_i \in B_i(x_i, r_i)} f_i(\lambda, x_i, y_i)$ for the set of locally optimal plans of the i-th AE for a given plan x_i and control λ;

$$T(\lambda, x, r) = \prod_{i \in I} T_i(\lambda, x_i, r_i);\quad D(\lambda, r) = \prod_{i \in I} D_i(\lambda, r_i);\quad X^c(s) = X(s) \cap \left(\prod_{i \in I} A_i(\lambda, s_i)\right)$$

is the set of coordinated plans $\prod_{i \in I} A_i(\lambda, s_i)$, of the AE that satisfy the restrictions X(s) of the AS. We consider the guaranteed value of the goal function of the AS on the set of locally optimal realizations of the AE for a choice of estimates $s = \{s_i\}$: $\Psi(\lambda(s), x(s), r) = \min_{y \in T(\lambda(s), x(s), r)} \Phi(\lambda(s), x(s), y)$.

In an AS with functioning mechanism $\Sigma^* = \langle W, B, \pi_{abc} \rangle$ in the equilibrium situation $s^* = r$ the value of $\Psi(\lambda(s), x(s), r)$ will be

$$\Psi(\lambda(s^*), x(s^*), r) = \max_{\lambda \in L,\, x \in X^c(r)} \Psi(\lambda, x, r) = \max_{\lambda \in L,\, x \in X^c(r)} \min_{y \in T(\lambda, x, r)} \Phi(\lambda, x, y) =$$

$$= \max_{\lambda \in L,\, x \in X^c(r)} \min_{y = x} \Phi(\lambda, x, y) = \max_{\lambda \in L,\, y \in Y(r) \cap D(\lambda, r)} \Phi(\lambda, y, y).$$

In this notation the following conditions are used. We remark that the guaranteed value $\Psi(\lambda(s), x(s), r)$ of the goal function of the AS on the set of locally optimal realizations of the AE in the situation $s = r$ is equal to the value of $\Psi(\lambda(s), x(s), r)$ at the optimal plan of the problem of coordinated planning (A.4)-(A.6) in [1] for $s = r$. This allows us to write the first inequality from the left. The second equality in this chain follows from the definition of the function $\Psi(\lambda(s), x(s), r)$. In writing the third equality we use the fact that, by Lemma 2, $\Sigma^* = \langle W, B, \pi_{abc} \rangle \in G^1$ and, consequently, by Lemma 3, in an equilibrium situation the set of locally optimal realizations of each AE has the form $T_i(\lambda, x_i, r_i) = x_i(s)$, $i \in I$. The fourth equality is based on Lemma 1.

We now consider any control law $\pi(s) \in G^1(W, B)$. Let s_π^* be the equilibrium situation (6) for this control law. In the equilibrium situation s_π^* the following conditions must hold:

$$\lambda_\pi^* = \lambda(s_\pi^*) \in L, \tag{A.12}$$

$$x_i(s_\pi^*) = y_{i_\pi}^*, \quad i \in I, \tag{A.13}$$

$$y_{i_\pi}^* \in D_i(\lambda, r_i), \quad i \in I, \tag{A.14}$$

$$y_\pi^* \in Y(r). \tag{A.15}$$

The conditions (A.12) and (A.15) are obvious, and the conditions (A.13) and (A.14) were proved in Lemmas 1 and 3. Considering (A.13), the value of the goal function of the AS in the equilibrium situation s_π^* is equal to $\Phi(\lambda_\pi^*, y_\pi^*, y_\pi^*)$, where λ_π^* and y_π^* satisfy the conditions (A.12), (A.14), and (A.15). Consequently, an upper estimate of the value of the goal function of the AS in the equilibrium situation s_π^* will be

$$\max_{\lambda \in L,\, y \in Y(r) \cap D(\lambda, r)} \Phi(\lambda, y, y) \geqslant \max_{\lambda_\pi^* \in L,\, y_\pi^* \in Y(r) \cap D(\lambda_\pi^*, r)} \Phi(\lambda_\pi^*, y_\pi^*, y_\pi^*)$$

and it coincides with the value of the goal function of the AS in the equilibrium situation $s^* = r$ for $\Sigma = \langle W, B, \pi_{abc} \rangle$ (A.11). Thus, $\forall\, \pi \in G^1(W, B)$ the value of the goal function in the equilibrium situation s_π^* does not exceed the value of the goal function of the AS in the equilibrium situation $s^* = r$ for above-board control laws. From this we get the analogous assertion with respect to the effectivity criterion of the C. With this the theorem is proved.

Proof of Theorem 3. Under the conditions of the theorem, for the value of the effectivity criterion of the C (12) in [2] in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ to be equal to the absolutely optimal value $K_\Sigma = 1$ it is necessary and sufficient that in the solution $\hat{s} = r$ of the game the following condition holds ((11 in [2]): $\Psi(\lambda(\hat{s}), x(\hat{s}), r) = \Psi_m(r)$. This holds, by the following chain of equalities:

$$\Psi(\lambda(\hat{s}), x(\hat{s}), r) = \max_{\lambda \in L,\, x \in X^c(r)} \Psi(\lambda, x, r) = \max_{\lambda \in L,\, x \in X^c(r),\, y = x} \Psi(\lambda, x, r) = \max_{\lambda \in L,\, x \in X(r),\, y = x} \Psi(\lambda, x, r) = \Psi_m(r).$$

The first equality defines the value of the function $\Psi(\lambda(\hat{s}), x(s), r)$ in the situation $\hat{s} = r$ in an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ (see (A.11)), the second equality can be written by virtue of the corollary to Lemma 3, and the third can be written by the epicoordination of the functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$. This proves the theorem.

Proof of Theorem 4. Suppose that the conditions of the theorem are satisfied in the AS. Then, by Theorem 1, the situation $s^* = r$ is an equilibrium in the sense of (6). We assume that in the AS there also exists another equilibrium situation s^* (6) such that $s^* \neq r$. In an AS with functioning mechanism $\Sigma = \langle W, B, \pi_{abc} \rangle$ in the equilibrium situation s^* the equilibrium conditions (6) must actually hold, or, in other notation, $x_i(s) \in A_i(\lambda(s^*), r_i)$, $i \in I$, as well as the conditions of perfect coordination (A.6) in [1]: $x_i(s^*) \in A_i(\lambda(s^*), s_i^*)$, $i \in I$. From this it follows that in the equilibrium situation s^* the following conditions must hold: $x_i(s^*) \in A_i(\lambda(s^*), r_i) \cap A_i(\lambda(s^*), s_i^*)$, $i \in I$. Since, by assumption, $s^* \neq r$, it follows that $\exists i_1 : s_{i_1}^* \neq r_{i_1}$, and for i_1 we can write $x_{i_1}(s^*) \in A_{i_1}(\lambda(s^*), r_{i_1}) \cap A_{i_1}(\lambda(s^*)), s_{i_1}^*) = \phi$. By the same token, there does not exists an equilibrium situation s^* (6) such that $s^* \neq r$. The theorem is proved.

Proof of Theorem 5. Suppose that the conditions of the theorem are satisfied in the AS. Then, by Theorem 1 and the properties of the goal functions of the AE with penalties for distortion of information (16) and (18) in [2], the situation $s^* = r$ is an equilibrium in the sense of (6). We assume that in the AS there also exists another equilibrium situation s^* (6) such that $s^* \neq r$. Then $\exists i_1 : s_{i_1}^* \neq r_{i_1}$. We use the notation $s^{**} = (s_1^*, \ldots, s^*_{i_1 - 1}, r_{i_1}, s^*_{i_1+1}, \ldots, s_n^*)$. We consider the effectivity criterion that takes into account the rule of choice of a realization of the AE with number i_1 in an AS with penalties for distortion of information (17) in [2]. We have the following chain of conditions:

$$\varphi_{i_1}^{pd}(\lambda^*, x_{i_1}(s^{**}), r_{i_1}, r_{i_1}, \chi_{i_1}(r_{i_1}, r_{i_1})) = \varphi_{i_1}(\lambda^*, x_{i_1}(s^{**}), r_i)$$

$$= \max_{x_{i_1} \in X_{i_1}(r_{i_1})} \varphi_{i_1}(\lambda^*, x_{i_1}, r_{i_1}) \geqslant \varphi_{i_1}(\lambda^*, x_{i_1}(s^*), r_{i_1}) > \varphi_{i_1}^{pd}(\lambda^*, x_{i_1}(s^*), r_{i_1}, s_{i_1}^*, \chi_{i_1}(s_{i_1}^*, \theta_{i_1}^*)). \tag{A.16}$$

Here $\lambda^* = \lambda(s^*)$, $\theta_{i_1} = \sigma_{i_1}(s_{i_1}^*, y_{i_1}^*) \neq s_{i_1}^*$. In this chain the first equality for the situation s^{**} holds by the condition (18) in [2], the next equality holds by the conditions of perfect coordination in the situation s^{**} (A.6) in [1], the next inequality holds by Lemma 1, and the last strict inequality for the situation s^* holds by the condition (18) in [2]. Thus,

$$\varphi_{i_1}^{pd}(\lambda^*, x_{i_1}(s^*), r_{i_1}, s_{i_1}^*, \chi_{i_1}(s_{i_1}^*, \theta_{i_1})) < \varphi_{i_1}^{pd}(\lambda^*, x_{i_1}(s^{**}), r_{i_1}, r_{i_1}, \chi_{i_1}(r_{i_1}, r_{i_1})),$$

which contradicts the fact that s^* is an equilibrium situation. This contradiction proves the theorem.

Remark. As can be observed in the course of the corresponding proofs, the foregoing results can be carried over also to the case of functioning mechanisms of an AS with partial planning of the realizations of the elements and the goal functions of the elements without penalties for deviation from the plan in a number of components of the realization vectors of the elements. Here it is only necessary to consider that in those cases when we are dealing with the coincidence (or disagreement) of the plan and the realization or with the analogous conditions for the set of plans and realizations, it is necessary to have in mind the plans x_i and the corresponding planned components of the realizations $y_i p$ of the AE, $i \in I$.

LITERATURE CITED

1. V. N. Burkov and V. V. Kondrat'ev, "Two-level active systems. I. Basic concepts and definitions," Avtom. Telemekh., No. 6, 64 (1977).
2. V. N. Burkov and V. V. Kondrat'ev, "Two-level active systems. II," Avtom. Telemekh., No. 7, 62 (1977).
3. J. Nash, "Coalitionless games," in: Matrix Games [Russian translation], Fizmatgiz, Moscow (1961), pp. 60-69.
4. Yu. B. Germaier, I. A. Vatel', F. I. Ereshko, and A. F. Kononenko, "Games with nonconflicting interests," in: Proc. All-Union School Seminar on Control of Large Systems [in Russian], Mitsniereba (1973), pp. 88-136.
5. V. S. Nemchinov, "Socialistic economy and planning of production," Communist, No. 5, 40 (1964).
6. M. Intrilligator, Mathematical Optimization and Economic Theory, Prentice-Hall, New Jersey (1971).
7. V. N. Burkov and V. I. Opoitsev, "The metagame approach to control of hierarchical systems," Avtom. Telemekh., No. 1, 103 (1974).
8. V. N. Burkov and V. V. Kondrat'ev, "Quasi-optimality of the principle of above-board control in the problem of allocation of resources," in: Proc. All-Union Conference on Control [in Russian], Nauka, Moscow (1974), pp. 357-360.
9. V. N. Burkov and V. V. Kondrat'ev, "Quasi-optimality of openhanded control in resource allocation," Preprints International Federation of Automatic Control 6th Triennial World Congress, Part IIID, Paper No. 41.5, Instrument Society of America, Pittsburgh (1975).
10. S. V. Emel'yanov and V. N. Burkov, "Control of active systems," in: Active Systems [in Russian], IAT, Moscow (1973), pp. 3-39.
11. S. V. Emel'yanov and V. N. Burkov, "Theory of active systems," (survey) in: Coordinated Control [in Russian], IAT, Moscow (1975), pp. 3-39.
12. V. N. Burkov and V. V. Kondrat'ev, "An analysis of the laws of above-board control of active systems from the position of the principle of guaranteed result," in: Trudy MFTI, Ser. Radiotekhnika i Élektronika, No. 9, Izd. MFTI, Moscow (1975), pp. 18-25.