HIERARCHY OPTIMIZATION: THEORY AND APPLICATIONS

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Abstract. We survey the body of analytical and numerical methods for hierarchy optimization developed during the recent decade and sketch their recent applications.

The problems of hierarchy optimization arise in different areas, from computer science to management. The well-known examples include the problems of an optimal prefix code, decision tree growing, communication network optimization, etc. They are studied separately, although allowing for a uniform description in terms of a general mathematical framework. This framework provides a common language to model different problems of hierarchy optimization, develop universal solution methods, and adopt and generalize local approaches.

An hierarchy optimization problem is to minimize a cost function by the choice of an admissible hierarchy (usually, the set of admissible hierarchies is too large for exhaustive search to be possible). The concepts of the abstract hierarchy and the sectional cost function suggested by Voronin and Mishin in 2001 provide a basis for viable study. The hierarchy is understood as an acyclic graph (generally, not a tree) with the given set of leafs and the sole root. The set of leafs is fixed by the problem setting, be it an alphabet in the coding problem or a set of base stations in telecom. The hierarchy cost function is called sectional if it sums up the costs of hierarchy nodes, while the node cost depends on the tuple of groups of leaves reachable from the children of this node.

Analytical methods of hierarchy optimization include conditions to narrow the set of potential solutions, and estimates of the attributes of the optimal hierarchy. Numeric methods reduce to exact and approximate algorithms for optimal hierarchy search. Analytical estimates of hierarchy cost are intensively used in algorithms to branch and cut the search space.

Sectional cost functions cover a wide range of applications, but are concise enough to allow for comprehensive deductions about the optimal hierarchy shape - when a hierarchy is tall or flat, tree-shaped, or resembles a conveyor belt. Also numerous algorithms were developed by Mishin to seek an optimal hierarchy, a tree, or a conveyor for sectional cost functions.

Nevertheless, the optimal hierarchy problem for a sectional cost function has no efficient solution in general. Homogenous cost functions provide an interesting subclass, which allows for a complete solution of an optimal hierarchy problem. The optimal hierarchy is proved to be uniform, the closed-form solution is derived for optimal hierarchy cost and shape (span of control and skewness), and efficient algorithms were developed to construct nearly-optimal hierarchies.

Key words and phrases. hierarchy optimization, growing decision tree, user menu design, hierarchy in firms, assembly line balancing.

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The suite of optimization techniques developed allowed solving the following hierarchy optimization problems in different areas.

In data mining - for the problem of growing a decision tree (used in classification and machine learning) a new combinatory lower-bound estimate for classification costs is suggested along with new efficient algorithms for building a decision tree.

In human-computer interaction - for the problem of hierarchical menu design a mathematical model was proposed for menu structure optimization and efficient algorithms were developed and implemented in the convenient computer-based design tool.

In management - the models of a management hierarchy were explored to address the fundamental issues of the theory of the firm. The closed-form expressions are obtained for hierarchy maintenance costs, span of control, the number of layers, managers' efforts and compensation, which enable comparative static analysis and model identification from data.

In production planning - an assembly line balancing problem was reduced to a special case of a hierarchy optimization problem; new algorithms were suggested for the generalized setting.

In the report we discuss the latest results, recent applications and perspectives. Potential applications of the theory and techniques are very diverse. To name a few: for information theory - handy taxonomies and classifiers, efficient schemes for hierarchical calculus; for business administration - methods for supply networks structure optimization.

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