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6.3 Flutter Mode Control

This is a technique for actively damping structural flutter modes, using aerodynamic control surfaces. This concept is mainly used for providing structural weight savings and for extending the flutter speed envelope to lower altitudes of higher Mach numbers.

A crude explanation of a flutter control system is given in Fig. 14. The flutter phenomenon for an elastic wing is due to a coupling between wing deformation (bending and torsion) and the aerodynamic unsteady forces induced by such deformations. These forces increase with kinetic pressure (namely with increasing speed or with decreasing altitude). Finally, this coupling can induce a forced excitation in resonance with some structural modes, leading to negative damping and to wing destruction (Fig. 14a).

To control this flutter behavior, a wing flap is used to develop an aerodynamic force opposite to the structural motion (Fig. 14b). This movement is detected by wing accelerometer indications sent to a dedicated computer, which, through a specific control law, monitors an active aileron. A wind-tunnel validation of such an automatic control is shown in Fig. 14c, where an effective torsion-mode damping is maintained despite increasing Mach number. Without active control, a strong premature flutter, arises.

The safety impact of a flutter suppression system (FSS) is highly dependent upon the aircraft configuration mission:

(i) for a civil transport aircraft this system must be non-flight-critical for certification, and would be used to enhance flutter speed from the maximum operational (V_{MO}) to 20% above the dive speed ($1.2V_D$), for some structural weight saving, without danger;

(ii) for a combat aircraft, a flutter suppression system is much more interesting because the flutter-limited flight envelope is mainly due to the extensive installation of external loads under the wings (the clean configuration is usually flutter-free). Such a heavily loaded configuration is non-flight-critical because, in the case of a system failure, it is easy to automatically jettison the external loads (fuel tanks, armament, etc.).

7. Application of Active Control Technology to Rotorcraft

Conventional helicopter control is obtained through a mechanical system on the rotor hub, producing a monocyclic blade pitch variation. Two limitations arise with this system: the pitch variation is limited at one per revolution, and the periodical variations of all blades are the same. A multicyclic pitch variation through an active control system can overcome such limitations for reducing vibration level, eliminating blade stall, enhancing stability and handling quality.

This individual blade-contol concept offers the promise of using electrohydraulic actuators linked to each blade and installed directly on the rotating part of the rotor hub, instead of the conventional installation.

See also: Automatic Flight Control Systems: History; Wing Load Alleviation Modelling

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P. Poisson-Quinton

Active Goal-Seeking Systems

An active goal-seeking system is a model of a large-scale system, a model which includes active elements. An active element is an elementary model of an intelligent system (man, team, artificial intelligence system and so on) which acts in a purposeful way or is capable of forecasting the future; in addition, in pursuance of its objectives, it is able to make decisions in reporting the information to the control system (center) or other system elements and in the choice of its state.

The main object of study in active systems at present is their functioning mechanism. This incorporates ways in which information is formulated in the center on elements and the environment, planning procedures (decision making by the center on values of control parameters or planned targets), an incentive system (set of objective functions of elements) and the objective function of the center.

The basic goal in the theory of active systems is to determine the functioning mechanism which is maximally effective on a specified set of functioning mechanisms. The theory is applied to analysis and improvement of control and management in socioeconomic systems. At present, the theory of two-level active systems which are described by deterministic models is essentially developed.

1. Definition of a Two-Level Active System and Functioning Mechanism

1.1 Statement of the Problem

A two-level active system consists of the center and n active elements. Let y_i denote the state of the ith element,

$y = (y_1, y_2, \ldots, y_n)$ the state of the system, Y_i the set of possible states of the ith element and Y the set of possible states of the system. The functioning mechanism of an active system is a set of rules (procedures, functions) which prescribe the actions of the center and elements in the functioning. The component parts of the functioning mechanism include: the element incentive system, the center objective function, the planning law, the sequence of center and element actions in the functioning, and procedures for the formulation of data to be used by the center for obtaining information on the elements. Let us consider a sequence of actions taken by the center and elements in each functioning period.

At the stage of data formulation, the center develops an idea of element models either through processing the data on preceding functioning periods (the adaptive technique) or by obtaining the desired data directly from the elements (two-way technique). Let s_i denote information on the model of the ith element, $s = (s_1, s_2, \ldots, s_n)$. At present the basic results in the active systems theory are only available for the two-way technique. The reported information s_i is an estimate of the parameters r_i of the set $Y_i(r_i)$. The center is assumed to know the parametric representation $Y_i(r_i)$ and the set Ω_i of possible values of the parameters $r_i (r_i \in \Omega_i)$. The subsequent discussion will concentrate on this case.

At the planning stage the center specifies the planned target $\pi_i(s)$ for each element; consequently, the plan for the entire system is $\pi(s) = \{\pi_1(s), \ldots, \pi_n(s)\}$. At the stage of implementing the plan each element selects a state $\hat{y}_i \in Y_i$ and defines the value of the objective function $f_i(\pi_i, \hat{y}_i)$.

With a specified sequence of actions and way to formulate the data, the functioning mechanism is defined by the three-tuple $\Sigma = (\phi, w, \pi)$ where $w = (f_1, f_2, \ldots, f_n)$ is the incentive system (set of objective functions) and $\phi(\pi, y)$ the objective function of the center.

If the objective function of the center is fixed, the functioning mechanism is defined by π and w only $[\Sigma = (w, \pi)]$. Later discussion will concentrate on this case.

The functioning of an active system with a specified functioning mechanism Σ can be regarded in a natural way as a game of n persons (active elements). The strategy of the ith element is choice of the estimate to be reported to the center, $\hat{s}_i \in \Omega_i$, and, once the information on the plan $\pi_i(\hat{s})$ is received, choice of the state $\hat{y}_i \in Y_i(r_i)$ such that $\hat{y} \in Y(r)$. Let $R(\Sigma)$ be the solution, or the set of situations $(\hat{s}, \hat{y})$ which can be implemented in a way compatible with earlier assumptions on behavior of the elements.

The effectiveness of the functioning mechanism is defined as

$$K(\Sigma) = \min_{(\hat{s},\hat{y}) \in R(\Sigma)} \phi[\pi(\hat{s}), \hat{y}] \tag{1}$$

In any further discussion it will be assumed that estimates of all parameters of the vector r are reported and all components of the state y are planned. Extension to the case of reporting only some estimates and planning just some components is easy.

The value

$$\Delta(\Sigma) = \max_{(\hat{s},\hat{y}) \in R(\Sigma)} \| \hat{y} - \pi(\hat{s}) \| \tag{2}$$

characterizes the degree to which the planned target is met and the value

$$\delta(\Sigma) = \max_{(\hat{s},\hat{y}) \in R(\Sigma)} \| s - r \| \tag{3}$$

the credibility of the information from the elements. $K(\Sigma)$, $\Delta(\Sigma)$ and $\delta(\Sigma)$ are basic characteristics of the functioning mechanism Σ. The functioning mechanism Σ for which $\Delta(\Sigma) = 0$ and $\delta(\Sigma) = 0$, or $\hat{y} = \pi(s)$ and $\hat{s} = r$ is referred to as regular.

Assume that a certain set G of feasible functioning mechanisms is given. The basic problem in the active systems theory is in determining the functioning mechanism Σ which is maximally effective on the set G.

1.2 The Case of Full Knowledge

If $\Omega_i = r_i$, then the center has complete information on the sets $Y_i = Y_i(r_i)$. This is the most thoroughly explored part of the active systems theory. In this case no information S should be obtained from the elements and the element strategy is the only choice of the state y_i. Assume first the following assumption about the independent behavior of the elements: each ith element chooses a state y_i from the set

$$P_i(\pi_i) = \arg\max_{y_i \in Y_i} f_i(\pi_i, y_i), \quad P(\pi) = \prod_{i=1}^{n} P_i(\pi_i) \tag{4}$$

Denote $P = \cup_{\pi \in D} P(\pi)$, where D is the set of feasible plans. If $P(\pi) \subset Y$ then the active system is referred to as a system with independent elements. For such systems the assumption of independent behavior is a natural assumption on the behavior.

2. Optimal Design of Planning Law Equations

Consider the optimal design of functioning mechanisms with a specified objective function $\phi(\pi, y)$ of the center, incentive system w and set of feasible plans D. This problem is referred to as the design of an optimal planning law equation. It is clear that $R(\Sigma) = P(\pi)$ and

$$K(\Sigma) = K(\pi) = \min_{y \in P(\pi)} \phi(\pi, y) \tag{5}$$

From (5) it follows that the optimal planning law has an effectiveness

$$K(\pi^{\mathrm{OPF}}) = \max_{\pi \in D} \min_{y \in P(\pi)} \phi(\pi, y) \tag{6}$$

This is referred to as the law of optimal planning with state forecasting, OPF.

Let us take up the optimal design of a planning law on a set of regular mechanisms, or with a constraint

$\Delta(\Sigma) = 0$. We shall assume benevolence of the elements towards the center in that if $\pi_i \in P_i(\pi_i)$, then $\hat{y}_i = \pi_i$.

We represent the objective function of the ith element as

$$f_i(\pi_i, y_i) = h_i(y_i) - \chi_i(\pi_i, y_i) \tag{7}$$

where

$$h_i(y_i) = f_i(y_i, y_i), \chi_i(\pi_i, y_i) = f_i(y_i, y_i) - f_i(\pi_i, y_i) \geqslant 0$$

and, analogously, the objective function of the center as

$$\phi(\pi, y) = H(y) - \theta(\pi, y) \tag{8}$$

where

$$H(y) = \phi(y, y), \theta(\pi, y) = \phi(y, y) - \phi(\pi, y) \geqslant 0$$

Define the set of coordinated plans of the ith element

$$S_i = \{\pi_i : \pi_i \in P_i(\pi_i)\} \tag{9}$$

and, consequently, $S = \Pi_{i=1}^{n} S_i$ is the set of coordinated plans of the active system.

The set of plans D will be assumed here and later to be large enough so that $P \subset D$. The planning law π^{CCP}

$$K(\pi^{\text{CCP}}) = \max_{\pi \in S} H(\pi) \tag{10}$$

will be referred to as the law of completely coordinated planning, CCP. It is clear that $K(\pi^{\text{CCP}}) \leqslant K(\pi^{\text{OPF}})$. What then are the cases where $K(\pi^{\text{CCP}}) = K(\pi^{\text{OPF}})$, or where the optimal planning law is that of completely coordinated planning?

THEOREM 1. *If $S = P$, then $K(\pi^{\text{CCP}}) = K(\pi^{\text{OPF}})$. Since $S \subset P$, then when $S = P$, the functioning mechanism is referred to as maximally coordinated (M coordinated).*

The condition $S = P$ is difficult to verify. It would therefore be useful to find constraints on the incentive system which are sufficient for M coordination. These are provided by Theorem 2.

THEOREM 2. *For the incentive system $w_i = h_i(y_i) - \chi_i(\pi_i, y_i)$, $i = 1, 2, \ldots, n$ to be M coordinated, it is sufficient that $\forall \pi \in D$, $y, y' \in Y$ and the penalty functions $\chi_i(\pi_i, y_i)$ satisfy the "triangle inequality"*

$$\chi_i(\pi_i, y_i) \leqslant \chi_i(\pi_i, y_i') + \chi_i(y_i', y_i) \tag{11}$$

The incentive system whose penalty function satisfies the triangle inequality is referred to as strongly coordinated (S coordinated).

The following theorem indicates a number of useful properties of S coordinated incentive systems.

THEOREM 3. *Let the incentive systems $w^1 = (h^1, \chi^1)$ and $w^2 = (h^2, \chi^2)$ be S coordinated. Then the incentive system $w = (h, \chi)$, where $\chi_i = a_i \chi_i^1$, $a_i > 0$, or $\chi_i = \chi_i^1 + \chi_i^2$ or $\chi_i = \max(\chi_i^1, \chi_i^2)$ is also S coordinated.*

One kind of S-coordinated incentive system is one with separable penalty functions

$$\chi_i(\pi_i, y_i) = \sum_{j=1}^{m} \chi_{ij}(\Delta_{ij})$$

where $\Delta_{ij} = \pi_{ij} - y_{ij}$ and χ_{ij} are concave nondecreasing functions on the semi-axes $\Delta_{ij} > 0$ and $\Delta_{ij} < 0$.

Another important example is systems with penalty functions

$$\chi_i(\pi_i, y_i) = \begin{cases} 0, & y_i = \pi_i \\ \rho_i(y_i), & y_i \neq \pi_i \end{cases} \tag{12}$$

Such penalty functions are referred to as plan-independent penalties (PIN functions). For incentive systems with such penalties, the conditions which dictate the set of completely coordinated plans are easily defined. For this purpose determine

$$b_i = \max_{y_i \in Y_i} [h_i(y_i) - \rho_i(y_i)]$$

The set S_i of completely coordinated plans of the ith element is given by the inequality

$$S_i = \{\pi_i \in Y_i : h_i(\pi_i) \geqslant b_i\} \tag{13}$$

Consequently, the law of completely coordinated planning has the form

$$\max_{\pi \in Y} H(\pi)$$

$$h_i(\pi_i) \geqslant b_i, \quad i = 1, 2, \ldots, n \tag{14}$$

3. The Degree of Centralization and Relative Effectiveness of Incentive Systems

Let us have a look at two incentive systems for the ith element which feature the same functions $h_i(y_i)$ and differ only in the penalty functions

$$w_i^1 = h_i(y_i) - \chi_i^1(\pi_i, y_i)$$

$$w_i^2 = h_i(y_i) - \chi_i^2(\pi_i, y_i)$$

Let us assume that the incentive system w_i^2 has a large higher degree of centralization that w_i^1 does ($w_i^2 > w_i^1$) if $w_i^2 \not\equiv w_i^1$ and for all $\pi_i \in D_i$, $y_i \in Y_i$ it is true that

$$\chi_i^2(\pi_i, y_i) \geqslant \chi_i^1(\pi_i, y_i) \tag{15}$$

In a similar way let us assume that the functioning mechanism Σ^2 has a higher degree of centralization for the ith element than $\Sigma^1 (\Sigma_i^2 > \Sigma_i^1)$ if $w_i^2 > w_i^1$. If $\Sigma_i^2 > \Sigma_i^1$ for all elements, then let us assume that the mechanism Σ^2 has a higher degree of centralization ($\Sigma^2 > \Sigma^1$).

A minimal degree of centralization is inherent in "market place mechanisms" for which $\chi_i(\pi_i, y_i) = 0$ with all $\pi_i \in D_i$, $y_i \in Y_i$ while the maximal one is featured by "mechanisms with strong penalties" for which

$$\chi_i(\pi_i, y_i) \geqslant L$$

where L is a sufficiently large number, with all $\pi_i \in D_i$, $y_i \in Y_i$. Mechanisms with strong penalties are associated with complete planning centralization whereby any feasible plan is found to be fulfilled. Incentive systems with strong penalties are obviously M coordinated (but not necessarily S coordinated) because

$S = P = Y$. The OPF and CCP laws then coincide and take the form of an optimal planning (OP) law

$$K(\pi^{OP}) = \max_{\pi \in Y} H(\pi) \tag{16}$$

THEOREM 4. *Let $\Sigma^2 > \Sigma^1$ and S^2 and S^1 be associated sets of completely coordinated plans. Then $S^1 \subset S^2$.*

From Theorem 4 it follows that if $\Sigma^1 = (\pi, w^1)$ is a regular mechanism $[\Delta(\Sigma^1) = 0]$, then $\Sigma^2 = (\pi, w^2)$ is also a regular mechanism. In other words, the regularity is preserved as the centralization degree increases.

Let us consider regular functioning mechanisms and for each incentive system w determine the associated law of coordinated planning $\pi^{CCP}(w)$. The efficiency $K(w)$ of the incentive system w is the efficiency of associated regular mechanism $\Sigma = (w, \pi^{CCP})$. From Theorem 4 it follows that if $w^2 > w^1$, then $K(w^2) \geqslant K(w^1)$ or that with increasing centralization degree, the efficiency of the incentive system does not decrease.

If nonregular mechanisms are also studied, then the efficiency of w is understood as that of the mechanism $\Sigma = [w, \pi^{OPF}(w)]$, that is, the incentive system w and the associated law of optimal planning with forecast of the state. In this case the conclusion that a system with a higher centralization degree is at least equally efficient is no longer true. The sufficient conditions for at least equal effectiveness of the incentive system w^2 in comparison with those of w^1 look more complicated.

THEOREM 5. *If for all $\pi \in D$, $y', y \in Y$, $y' \neq y$ it is true that*

$$\begin{aligned} &h^2(y') - h^2(y) + \chi^2(y', y) \\ &\qquad \geqslant h^1(y') - h^1(y) + \chi^1(\pi, y) - \chi^1(\pi, y') \end{aligned} \tag{17}$$

then w^2 is no less efficient than w^1.

In particular, for systems of comparable centralization degree $h^1 = h^2$, the condition of Theorems takes a simpler form

$$\chi^2(y', y) \geqslant \chi^1(\pi, y) - \chi^1(\pi, y').$$

Note that if w^1 is an M-coordinated system, then for an at least equal efficiency of w^2 it would be sufficient if $w^2 > w_1$. Finally, if χ^2 are PIN penalties, then for an at least equal efficiency of w^2 it would be sufficient if $w^2 > w^1$.

4. Design of Incentive Systems

Assume that a set G_w of possible incentive systems $w = (h, \chi)$ is given. For optimal design of incentive systems it is required to determine on that set an incentive system which is maximally efficient [with an appropriate law $\pi^{OPF}(w)$]. The set G_w is usually specified as certain requirements to the incentive system such as regularity of the functioning mechanism or constrained reward and penalty. Thus, real-life constraints on penalties, with specified functions $h_i(y_i)$, can be

$$\chi_i^{min}(\pi_i, y_i) \leqslant \chi_i(\pi_i, y_i) \leqslant \chi_i^{max}(\pi_i, y_i) \tag{18}$$

for all $\pi \in D$ and $y \in Y$.

If a regular functioning mechanism is to be obtained, then from Theorem 4 it follows that the optimal system is (h, χ^{max}) with a maximal centralization degree. If there is no need for the functioning mechanism to be regular, then a solution can only be obtained in particular cases. Thus, if G_w is defined by inequalities

$$0 \leqslant \chi_i(\pi_i, y_i) \leqslant \rho_i(y_i)$$

for all $\pi \in D$ and $y \in Y$, then, by virtue of the corollary of Theorem 5, the incentive system (h, ρ) which features a maximal centralization degree is also optimal.

Finally, take up design of incentive systems where constraints are imposed on the functions $f_i(\pi_i, y_i)$ as entities rather than on penalty functions

$$f_i^{min}(\pi_i, y_i) \leqslant f_i(\pi_i, y_i) \leqslant f_i^{max}(\pi_i, y_i) \tag{19}$$

Denote

$$f_i^*(\pi_i, y_i) = \begin{cases} f_i^{max}(\pi_i, y_i) & \text{if} \quad y_i = \pi_i \\ f_i^{min}(\pi_i, y_i) & \text{if} \quad y_i \neq \pi_i \end{cases} \tag{20}$$

We shall now determine the cases in which the incentive system $w^* = \{f_i^*\}$ (maximal reward with implementation of the plan and minimal in the case of deflection) is the optimal solution of the design problem on the set G_w of incentive systems defined by condition (19). Let the boundary functions f_i^{min} and f_i^{max} meet the conditions: for all $\pi \in D$, $y \in Y$

$$f_i^{max}(y_i, y_i) \geqslant f_i^{max}(\pi_i, y_i)$$
$$f_i^{min}(y_i, y_i) \geqslant f_i^{min}(\pi_i, y_i)$$

THEOREM 6. *The incentive system w^* is optimal on the set G_w (19) if for all $\pi \in D$, $y, y' \in Y$, $y \neq y'$ it is true that*

$$\begin{aligned} &f_i^{max}(y_i', y_i') - f_i^{max}(\pi_i, y_i') \\ &\qquad \geqslant f_i^{min}(y_i', y_i) - f_i^{min}(\pi_i, y_i) \end{aligned} \tag{21}$$

or, denoting

$$\chi_i^{max}(\pi_i, y_i) = f_i^{max}(y_i, y_i) - f_i^{max}(\pi_i, y_i)$$
$$\chi_i^{min}(\pi_i, y_i) = f_i^{max}(y_i, y_i) - f_i^{min}(\pi_i, y_i)$$

the conditions (21) can be written in a simpler equivalent form

$$\chi_i^{max}(\pi_i, y_i') \geqslant \chi_i^{min}(\pi_i, y_i) - \chi_i^{min}(y_i', y_i) \tag{22}$$

5. Systems with Dependent Elements

If $P(\pi) \in Y$, then the assumption of independent behavior does not hold since with independent choice it may be found that $y \notin Y$. In this case we have a system with dependent elements. Definition of the solution to the game becomes more involved. We can recognize the interdependence of the elements in their objective functions assuming that

$$\chi_i(\pi_i, \hat{y}_i) = L \quad \text{if} \quad \hat{y} \notin Y$$

where L is a sufficiently large number such that choice of any implementable state $\hat{y} \in Y$ is more preferable for all elements than that of any nonimplementable state $\hat{y} \notin Y$.

Now we can confine ourselves to local constraints on the choice of element states because global constraints are allowed for in objective functions of elements. The objective functions are now, however, dependent on states of all elements $f_i(\pi_i, y)$ rather than on the state of a given element. Dependence of objective functions of elements on plans and states of other elements can also be felt because of the limited total incentive fund N. In this case, the fund N is allocated directly proportionally with the values of $f_i(\pi_i, y_i)$ and the gain of the ith element is

$$f_i^N(\pi, y) = \frac{f_i(\pi_i, y_i)}{\sum_{j=1}^{n} f_i(\pi_j, y_j)} N$$

In this case the objective function of the ith element is dependent on the plan π and the state y of the entire system. To determine the solution to the game, denote as $\Pi_i(y_i)$ the set of possible values of the situation, $y(i) = (y_1, y_2, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$, or states of the other elements assumed by the ith element. The conditional guaranteed result of the ith element [the result which is ensured if its assumptions are true and indeed $\hat{y}(i) \in \Pi_i(y_i)$] is defined in the following way

$$f_i^{\Pi}(\pi, y) = \min_{y(i) \in \Pi_i(y_i)} f_i(\pi, y) \tag{23}$$

The function $f_i^{\Pi}(\pi, y_i)$ is referred to as the Π-criterion. In compliance with the principle of the maximal conditional guaranteed result, the element chooses $\hat{y}_i$ so as to $\max f_i^{\Pi}(\pi, y)$, or

$$f_i^{\Pi}(\pi, \hat{y}_i) = \max_{y_i \in Y_i} \min_{y(i) \in \Pi_i(y_i)} f_i(\pi, y) \tag{24}$$

This principle of choice will be referred to as the principle of conditional maximin and the associated solution as the Π solution. If the assumptions of all elements have come true, or $\hat{y}(i) \in \Pi_i(\hat{y}_i)$ for all i then the totality of sets $\Pi(y) = \{\Pi_i(y_i)\}$ will be referred to as justified and the associated situation as the Π equilibrium. Particular cases of Π equilibrium are guaranteeing situations

$$[\Pi_i(y_i) = \prod_{j \neq i} Y_j]$$

and Nash equilibria y^*. In the latter case $\Pi_i(y_i) = y^*(i)$ and

$$f_i(\pi, y^*) = \max_{y_i \in Y_i} f_i[\pi, y^*(i), y_i]$$

These definitions of a Π solution and Π equilibrium allow many results that were obtained for systems with independent elements to be extended to systems with dependent elements. Assume that the states of other elements dictate only the function $h_i(y)$ of the element, or

$$f_i(\pi_i, y_i) = h_i(y) - \chi_i(\pi_i, y_i) \tag{25}$$

The associated Π criterion has the form

$$f_i^{\Pi}(\pi_i, y_i) = h_i^{\Pi}(y_i) - \chi_i(\pi_i, y_i) \tag{26}$$

Now the study can be conducted as in the case of systems with independent elements. Consequently, the concept of M-coordinated and S-coordinated incentive systems is naturally extended. Thus if the penalty function $\chi_i(\pi_i, y_i)$ satisfies the triangle inequality, then the system is S coordinated with any sets $\Pi(y)$. The form of the problem of optimally completely coordinated planning is significantly dependent on the totality of sets $\Pi(y)$.

For regular functioning mechanisms, it is assumed that the plans of other elements will be met, or that $\Pi_i(y_i) = \pi(i)$. In this case

$$f_i^{\Pi}(\pi_i, y_i) = h_i[\pi(i), y_i] - \chi_i(\pi_i, y_i) \tag{27}$$

If the element does not rely on other elements meeting their planned targets, a more cautious assumption should be made

$$\| y(i) - \pi(i) \| \leqslant \eta_i \tag{28}$$

where η_i characterize the maximal possible deflection of the real situation from the planned one, in the light of the ith element.

Note that any totality of the sets $\{\Pi_i\}$ such that $\pi(i) \in \Pi_i$ is justified for a regular mechanism and, consequently, dictates the Π equilibrium. If $\Pi_i = \pi(i)$ then this equilibrium is also a Nash equilibrium.

As an example, let us consider a system with constant penalties

$$\chi_i(\pi_i, y_i) = \begin{cases} 0, & y_i = \pi_i \\ c_i, & y_i \neq \pi_i \end{cases}$$

where $c_i > 0$ is the penalty for deflection from the plan.

For such incentive systems the problem of completely coordinated planning can be represented in the form

$$\max_{\pi \in D} H(\pi)$$

$$h_i^{\Pi}(\pi_i) \geqslant b_i^{\Pi}, \quad i = 1, 2, \ldots, n \tag{29}$$

where

$$b_i^{\Pi} = \max_{y_i \in Y_i} h_i^{\Pi}(y_i) - c_i$$

The dependence of the set of completely coordinated plans on the totality of sets Π is clear. Thus if $\Pi = \pi(i)$, the set of completely coordinated plans will be given by the conditions

$$h_i(\pi) + c_i \geqslant \max_{y_i \in Y_i} h_i[\pi(i), y_i]. \tag{30}$$

If, however, Π_i is determined by (28), then the set of completely coordinated plans is defined by quite different conditions

$$\min_{\|y(i)-x(i)\|\leqslant\eta_i} h_i[y(i),\pi_i]+c_i \geqslant \max_{y_i\in Y_i}\ \min_{\|y(i)-x(i)\|\leqslant\eta_i} h_i[y(i),y_i] \quad (31)$$

6. *Laws of Coordinated Planning*

The underlying principle of laws of completely coordinated planning is that meeting the planned targets should be beneficial for the elements. This, moreover, is not the sole possible way of coordinating the plans with the interests of elements. Thus the coordination of interests can be obtained by, for example, making plans which are most beneficial for the elements or those which ensure obtaining certain values of the objective functions, or plans such that any deflection is certain to stay within a set range.

It is therefore desirable to develop a sufficiently general planning procedure which incorporates the idea of coordinating the plans with the interests of the elements. This procedure is referred to as the principle of coordinated planning. Laws of coordinated planning are constructed by completing the problem of optimal planning

$$\max_{\pi\in D} H(\pi) \quad (32)$$

with so-called conditions of coordinated planning

$$\psi_i(\pi_i)=\max_{z_i\in D_i}\psi_i(z_i), \quad i=1,2,\ldots,n \quad (33)$$

In coordinated planning the functions $\psi_i(\pi_i)$ are referred to as element preference functions. By specifying these in different ways different laws of coordinated planning can be obtained. Thus the law of completely coordinated planning is obtained if

$$\psi_i(\pi_i)=1[\varphi_i(\pi_i)-h_i(\pi_i)] \quad (34)$$

where

$$\varphi_i(\pi_i)=\max_{y_i\in Y_i} f_i(\pi_i,y_i), \quad 1[x]=1 \quad \text{if} \quad x\geqslant 0$$

and is equal to zero if $x<0$.

If $\psi_i(\pi_i)=\varphi_i(\pi_i)$, then the law of "fair play" control is obtained.

Laws of ϵ-coordinated planning are associated with the choice

$$\psi_i(\pi_i)=1[\varphi_i(\pi_i)-\epsilon_i\max_{z_i\in Y_i}\varphi_i(z_i)], \quad 0<\epsilon_i<1 \quad (35)$$

With this choice of preference functions the coordination conditions can be written in the form

$$\varphi_i(\pi_i)\geqslant\epsilon_i\max_{z_i\in Y_i}\varphi_i(z_i) \quad (36)$$

For completion of the case of complete knowledge, only the case of partial planning of state components should be considered. Study of this case is reduced to the above case of planning all components by forecasting the nonplannable components as functions of the plan and plannable components. Following substitution of forecast values (in the case of ambiguity the guaranteed result is taken) we obtain the case of planning all the components.

7. *Incomplete Knowledge on the Part of the Center*

In the case of incomplete knowledge, the center is unaware of accurate values of the parameters r_i which determine the set $Y_i(r_i)$ of possible states of the element but knows only a certain set Ω_i of possible values of r_i. Let us confine ourselves to the case of two-way data formulation and systems with independent elements. Note that following substitution into the element objective functions of the forecast states $y_i\in P_i(\pi_i)$ reduces the objective functions to the form

$$\varphi_i[\pi_i(s),r_i]=\max_{y_i\in Y_i(r_i)} f_i[\pi_i(s),y_i] \quad (37)$$

Now the system functioning can be regarded as a game of active elements whose strategies is reporting the estimates $s_i\in\Omega_i$.

In the optimal design of a functioning mechanism the following sequence is widely used. First the optimal mechanism $\Sigma(r)$ is developed while the center is assumed to be completely informed. Then the parameters r are replaced by the estimates s which are obtained from the elements. If for the resulting mechanism the reported estimates can be proved credible, or $s=r$ in the solution of the game, then the mechanism $\Sigma(s)$ is the solution of the optimal design problem. In this approach the basic point is to prove the credibility of the reported estimates. Credibility of the information from elements is the basic problem in the case of incomplete knowledge on the part of the center. The conditions which ensure credibility of the information are dependent on what is understood by the solution of a game of elements. Thus if the solutions are dominant strategies, then the necessary and sufficient credibility conditions can be obtained. Let Σ be a functioning mechanism with strong penalties and the set of possible plans of the ith active element $D_i(s)\neq\varnothing$ for all $s\in\Omega$ and is represented in the form

$$D_i(s)=Y_i(s_i)\cap X_i[s(i)] \quad (38)$$

where

$$s(i)=(s_1,s_2,\ldots,s_{i-1},s_{i+1},\ldots,s_n) \quad \text{and} \quad X_i[s(i)]$$

is an arbitrary fixed set. Then Theorem 7 holds.

THEOREM 7. *For the functioning mechanism $\Sigma=(w,\pi)$, where the data are formulated by a two-way technique, to report credible information as a dominant strategy with any $r\in\Omega$, it is necessary and sufficient that there exists $X_i[s(i)]$ such that*

$$\varphi_i[\pi_i(s),s_i]=\max_{z_i\in D_i(s)}\varphi_i(z_i,s_i), \quad i=1,2,\ldots,n \quad (39)$$

From Theorem 7 it follows that the optimal functioning mechanism on the set of functioning mechanisms which ensure reporting credible information as a dominant strategy should be sought on the set of functioning mechanisms with "fair play" control. The theorem assumes benevolence on the part of the elements vis-à-vis the center; in other words, an element is assumed to report a credible estimate if this is included in the set of dominant strategies.

Note that if some $\lambda_i(s)$ components of the plan $\pi_i(s) = [\lambda_i(s), x_i(s)]$ do not depend on the estimates s_i, or $\lambda_i(s) = \lambda_i[s(i)]$, then the condition (39) can be formulated for components $x_i(s)$ alone. This leads to a very important extension of "fair play" control laws to the case where $\lambda_i(s)$ are "weakly dependent" on the estimates of each element. (Thus, $\lambda(s)$ are product prices which were obtained from estimates given by all companies and are weakly dependent on any individual company.) The "weak impact" of the estimates s_i on the components $\lambda_i(s)$ leads to the assumption of weak effect whereby the elements neglect the effect of their estimates s_i on the components $\lambda_i(s)$.

Theorem 7 is also valid for arbitrary mechanisms if $D_i(s)$ is represented as $D_i(s) = B_i \cap X_i[s(i)]$, where B_i is no longer related to $Y_i(s_i)$ but is independent of s_i.

If the element strategy is required to be a guaranteeing or Nash equilibrium rather than dominant, then weaker conditions for the credibility of the information can be obtained.

Let us consider mechanisms Σ^ϵ with laws of ϵ-coordinated planning. Let $E_i(s_i, \epsilon_i)$ denote a set of coordinated plans of the ith active element. Assume that the ith element sticks to the principle of a maximal guaranteed result and this result is

$$\min_{\pi_i \in E_i(s_i, \epsilon_i)} \varphi_i(\pi_i, r_i)$$

Then Theorem 8 holds.

THEOREM 8. *For the reporting of credible information to be the only guaranteeing strategy of the ith element for any $r_i \in \Omega_i$ in an active system with a mechanism Σ^ϵ, it is necessary and sufficient that the following conditions of "equality of preference functions" hold*

$$E_i(s_i^1, \epsilon_i) \setminus E_i(s_i^2, \epsilon^i) \neq \varnothing \quad \text{for all} \quad s_i^1 \neq s_i^2 \qquad (40)$$

Finally, we shall look at a basic result for the case of a Nash equilibrium regarded as solution of the game.

THEOREM 9. *Let $\Sigma^2 > \Sigma^1$ and Σ^1 be a regular mechanism. Then Σ^2 is also a regular mechanism.*

This means that with increasing degree of centralization the properties of reporting credible information (in the case of a Nash equilibrium) and of meeting the planned targets are maintained. Hence it follows that with increasing degree of centralization the set of regular mechanisms does not reduce. Therefore if there are no constraints on the degree of centralization, the optimal regular mechanism should be sought on the set of mechanisms with complete planning centralization. this is equivalent to incentive systems with strong penalties.

Ensuring the reporting of credible information enables construction of optimal regular mechanisms under uncertainty. This requires meeting the conditions which ensure credibility of the information and conditions for completely coordinated plans with complete information available to the center. Combination of such conditions results in a wide range of situations in which optimal functioning mechanisms can be obtained under uncertainty.

8. *Lines of Further Research*

The above models of two-level active systems are very basic. Their extension proceeds along many lines such as recognition of uncertain factors in the model of the system and in the model of the environment, recognition of the model dynamics, and multilevel systems. Numerous other lines of research call for other schemata of active system functioning, such as functioning mechanisms of active systems that formulate data "with storage of information," adaptive control mechanisms in active systems and iterative planning schemata.

Theoretical studies in management of active systems are now supplemented with widespread applications of the methods and findings of the theory. The application range of the theory of active systems seems impressive. It includes analysis and improvement of management mechanisms in organizations at all levels of the national economy. The theory has been successfully applied to improvement of management mechanisms in industry, research institutions and sectors of the national economy, and early progress is being made in applying the theory to studies of socioeconomic systems, such as in public health and skill management.

See also: Goal-Oriented Systems

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