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AUTOMATIC CONTROL SYSTEMS

DECISION-MAKING MODELS FOR GENERALIZED ALTERNATIVE STOCHASTIC NETWORKS

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A new type of cyclic stochastic alternative time-oriented model (SATM) is suggested which makes it possible to account for both a wide range of relations between tasks (activities) and nodes (events) and the multialternative character of branching directions (lines) of implementation of a project. The available random actions, conditions, and noise find wide use in a network model. A general approach and a procedure are worked up for finding optimal solutions on the SATM. Decisions are made at deterministic (controllable) branching points (nodes), for which the choice of the direction of implementation of a project is taken by the manager of the project (stochastic branchings are not controllable). A quasioptimal solution is found which lies in the reduction of the SATM network to a simpler generalized alternative activity network (GAAN) model including both stochastic and deterministic branchings. For networks of the GAAN type, the solution of the problem for the choice of an optimal admissible version is known. On executing the available algorithm for defining an optimal version, we perform the inverse transformation of the obtained optimal subnetwork into a network of the SATM type. The derived results enable us to make an optimal decision in the course of the development of network projects of the most common type.

1. INTRODUCTION

In implementing network projects with multivariant outcomes (construction work, research or research and development work, etc.), the choice of an optimal version is of the utmost significance. In essence, this choice is equivalent to the optimal implementation of a project for a specified objective. Here, account is inevitably taken of risk factors, too, which is extremely important in control of projects.

In a number of the fields of application, the use of the available network models gives no way of making a justified decision in the act of implementing projects, although these projects actually take on a multivariant character. Thus, in construction engineering, for a long time use has been made of generalized network models [1] that employ a wide variety of logical links. However, these models do not admit the use of alternative branchings, although in a number of cases the models of construction engineering are multivariant in nature.

In addition, modern network models of control of projects must account in full measure for the presence of the indeterminacy factor. It is necessary to allow for the stochastic character of network models both in estimating the time (duration) it takes to perform individual tasks and in defining the structure of a network model on the whole. It should be kept in mind that the branching directions take on a stochastic character, which is not uncommon in multivariant network planning.

To solve the above-described problems for working out a more adaptable and adequate network model, we suggest a new type of stochastic alternative time-oriented model (SATM) that permits accounting for both a wide range of links between tasks and nodes and the multialternative character of branching directions of implementing a project. The existence of random actions, conditions, and noise find wide use in a network model.

In addition to the development of a new class of network models, we propose a general approach and a procedure intended to find optimal solutions on the SATM. The case in point is the decision-making at the

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deterministic (controllable) branching points, for which the choice of the line of development of a project is made by the manager of the project (stochastic branchings are not controllable).

In what follows, two branching deterministic nodes in a network model, together with the directions selected from them, will be called consistent (noncontradictory) if there is a possibility that one node falls within the other. In each network model there exists a finite set of the collections of consistent nodes, for which we can also select consistent directions of implementation. Each such collection implies in effect an admissible version of realization of the process. This kind of version contains branchings that take on only the stochastic character. If we select an optimal admissible version of the process realization with reference to some signs, parameters, or criteria, we are in a position to solve the problem for an optimal control of a multivariant network project. We prescribe a criterion K (an objective function of project execution) and constraints $O_1, O_2, \ldots, O_n$. If the selected admissible version gives an optimum to the criterion K and satisfies the preassigned constraints on other criteria, we are in a position to make an optimal decision at each deterministic branching point so as to continue implementing the project.

The difficulty involved with the stated problem lies in the complexity of constructing both one admissible collection of consistent nodes and all the admissible collections. It can be shown that the problem of the choice of an optimal collection is an **NP**-complete one, i.e., it can be solved by an exhaustive search. We suggest a quasioptimal solution that involves reducing the cyclic SATM to a simple cyclic generalized alternative activity network (GAAN) model [2] for which both stochastic and deterministic branchings also exist. For networks of the GAAN type, the solution of the problem of selecting an optimal admissible version is known [2, 3]. On executing the available algorithm for defining an optimal version, we perform the inverse transformation of the version into a network of the SATM type. We have to use an auxiliary GAAN model because the implementation of the complete collection of admissible versions in the class of SATM networks represents an extremely complex problem, for which we have not yet been able to find a solution.

The obtained results permit us to make optimal decisions in the course of execution of network projects of the most general kind. Let us note that in our opinion, the SATM is the most generalized of the available network models. The well-known models such as PERT, CPM, and GERT [4], CAAN [5] and GAAN [2, 3], GNM [1], etc. are special cases of the SATM.

2. DESCRIPTION OF THE SATM

In the course of its development, the SATM underwent a number of changes and modifications. Initially, the so-called generalized network model (GNM) [1] that did not have any alternative branchings and random parameters was worked out. At the second stage, the GNM was extended and supplemented by introducing the randomness and alternativeness.

The generalized network model features a large collection of branched logical-time links unavailable for other network models in existence. The GNM represents a finite oriented cyclic graph (with cycles of nonpositive length) consisting of arcs and nodes. The set of arcs is subdivided into arc-tasks and arc-links. The arcs of the first type implement a certain volume of production activity in time, and the arcs of the second type account solely for the logical links between the tasks with the use of various time constraints. Thus, the GNM is primarily oriented toward the arc-tasks and logical links between these arcs. Hence, we will denote the GNM by the symbol $N(A, L)$, where A identifies the set of active tasks and L stands for the logical links in time between the tasks. As for the nodes, they are scattered on the time axis of the tasks performed.

An example of the link between arbitrary points of two arc-tasks of the GNM appears in Fig. 1. The coordinate of a point on the arc is defined as a percentage of the volume of work. The link parameter can be both positive and negative. Examples of the network fragments admissible in the GNM are shown in Fig. 1.

We will denote a few basic kinds of nodes belonging to the tasks.

(1) $e_s(a)$ is a node belonging to the arc-task $a \in A$ and specifying the beginning of the performance of a.

(2) $e_f(a)$ is a node belonging to the arc-task $a \in A$ and specifying the completion of a.

(3) $e_r(a)$ is a node belonging to the arc-task $a \in A$ and lying between $e_s(a)$ and $e_f(a)$; generally, $e_r(a)$ specifies the process of completion of a definite preset volume of the task a, expressed as a percentage. Namely, nodes $e_r(a)$ (there can be a few such nodes) are used as input and output nodes on establishing the logical-time links between arc-tasks.

(4) $\{e(a)\} = e_s(a), e_{r_1}(a), \ldots, e_{r_q}(a), e_f(a)$ is the set of nodes e_A.

We introduce a number of additional designations.

(1) $\tau(a)$ is the deterministic time length of the completion of a task $a \in A$.

(2) $\overline{\tau}(a)$ is the lower bound of the value $\tau(a)$.

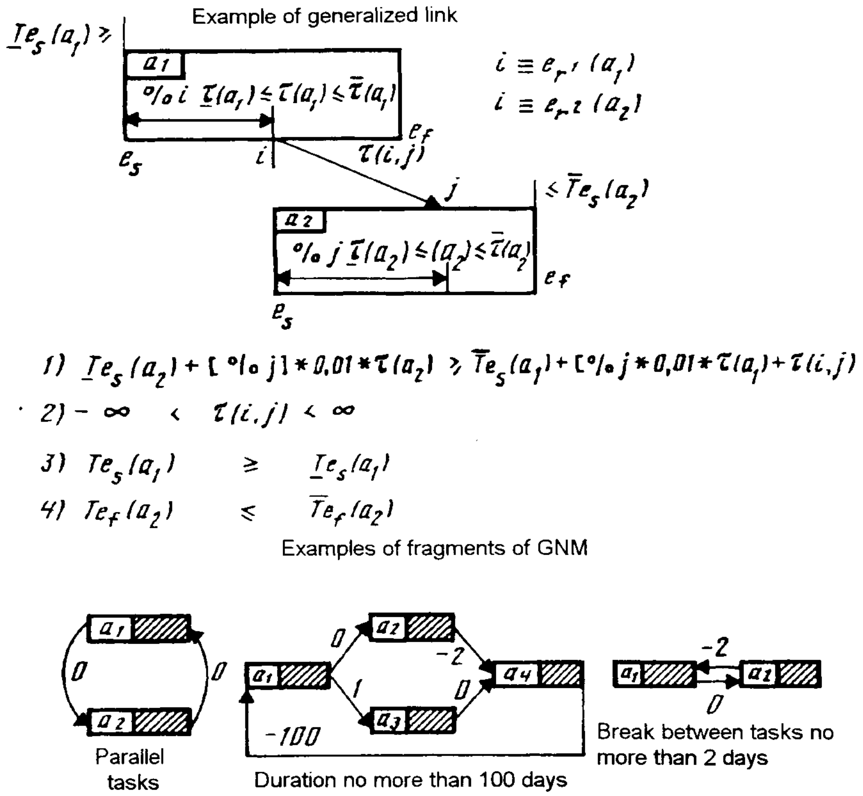

Fig. 1

(3) $\hat{\tau}(a)$ is the upper bound of the value $\tau(a)$. Note that the values $\overline{\tau}(a)$ and $\hat{\tau}(a)$ are preset for all tasks $a \in A$. Thus, there exists a constraint

$$\overline{\tau}(a) \leqslant \tau(a) \leqslant \hat{\tau}(a) \forall a : \; a \in A.$$

(4) $T_e(a)$ is the instant of the occurrence of one of the nodes e belonging to the arc-tasks a $(e_s(a), e_f(a)$, or one of $e_r(a))$.

(5) $\overline{T}_e(a)$ is the lower bound set on the instant of time $T_e(a)$.

(6) $\hat{T}_e(a)$ is the upper bound set on the instant of time $T_e(a)$. The bounds $\overline{T}_e(a)$ and $\hat{T}_e(a)$ take on a directional character. They can be prescribed for some $e \in a$ and for some $a \in A$.

(7) $T_{e_r}(a) - T_{e_s}(a) = 0.01 \cdot \tau(a) \cdot [\%e_r(a)]$ is the time interval from the beginning of carrying out a task a to a node $e_r(a)$, over which $[\%e_r(a)]$ of the volume of the task a will be done.

(8) $-\infty < \tau\{e_{rV}(a_1), e_{rW}(a_2)\} < \infty$ is the length of the arc connecting the nodes $e_{rV}(a_1)$ and $e_{rW}(a_2)$ (the arc extends from the first node and enters the second). Note that individual logical arcs can have negative lengths.

We will now consider examples of the logical links.

I. $[\%e_{rW}(a_2)]$ of the volume of the task a_2 must be done not earlier than in d units of time after the completion of $[\%e_{rV}(a_1)]$ of the task a_1.

The last constraint is written in the form of the relations

(a) $T_{e_s}(a_1) + \tau(a_1) \cdot 0.01 \cdot [\%e_{rV}(a_1)] + \tau\{e_{rV}(a_1), e_{rW}(a_2)\} \leqslant T_{e_s}(a_2) + \tau(a_2) \cdot 0.01[\%e_{rW}(a_2)]$;

(b) $\tau\{e_{rV}(a_1), e_{rW}(a_2)\} = d$.

II. Constraints on the beginning of the fulfillment of a task $a \in A$. There exists $T_{e_s}(a) \geqslant \overline{T}_{e_s}(a)$ or $T_{e_s}(a) \leqslant \hat{T}_{e_s}(a)$.

III. Constraints on the instant of realization of one node or a group of nodes within a task:

$$T_{e_{\xi V}}(a) \geqslant \overline{T}_{e_{\xi V}}(a), \quad 1 \leqslant v \leqslant q, \quad q \geqslant 1, \quad \text{or}$$

$$T_{e_{\eta V}}(a) \leqslant \hat{T}_{e_{\eta V}}(a), \quad 1 \leqslant v \leqslant \ell, \quad \ell \geqslant 1.$$

Let us note that all values of $\overline{T}_e$ and $\hat{T}_e$ take on a directional character and are preset. It should be kept in mind that, in the GNM, a task after its beginning is carried out without interruptions.

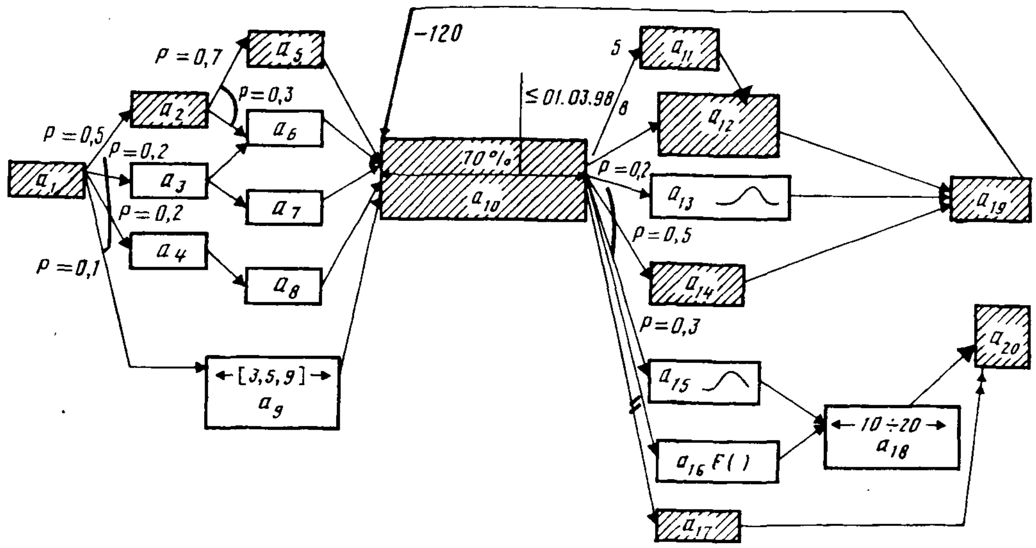

Fig. 2. A fragment of the stochastic alternative time-oriented model (SATM).

IV. Constraints on the time length (duration) of logical arcs:

$$-\infty < d_1 \leqslant \tau\{e_{r_V}(a_1), e_{r_W}(a_2) \leqslant d_2 < \infty\}, \ a_1, a_2 \in A.$$

V. Logical arcs with a negative time length.

If the complete duration of a task or a fragment of the GNM with input $e_s(a_V)$ and output $e_t(a_v)$ or $e_f(a_W)$ must not exceed d units of time, then there exists a relation $\tau\{e_f(a_W),\ e_S(a_V)\} = -d < 0$. The arcs of this type are said to be reverse.

In the course of the analysis of the GNM, the schedule of the occurrence of all nodes entering into the model is drawn up. The GNM bears the name of a consistent model if there exists at least one admissible schedule satisfying the above-described constraints (and a number of others). Otherwise, the network is inconsistent and cannot be implemented.

Figure 2 illustrates an extended fragment of the SATM with supplementary inclusions of various kinds of random actions and alternative branchings at the entry into or at the exit from various nodes and tasks. We will clarify this fragment:

The symbols $a_1 - a_{20}$ denote tasks.

The shaded regions of tasks illustrate one of the possible implementations of the SATM. The left edge of a task region denotes the beginning of performing the task and the right edge denotes the completion of the task.

The digits [3, 5, 9] denote the deterministic choice of the time length of a task out of the set 3, 5, 9. In a particular case, one digit designates a fixed time length.

The symbol ∫\ identifies a random value of the time length of a task with a specified distribution law (the normal, the β-distribution, the uniform law, etc.).

The symbol $F(\ )$ is the time length of a task as a function of the collection of arguments (for example, as a function of the output of the team or the job rate).

The numbers < 01.03.98 denote the node of "70% of fulfillment of a task" that must be completed by the 1st of March, 1998.

The digit 5 is the arc length, namely, 5 days.

The number −120 denotes the node j (accomplishment of the task a_{19}) that must occur no later than in 120 days after the occurrence of the node i (beginning of the task a_{10}).

The lines imply that a task begins after the implementation of both arcs (the logical "AND" operation).

The lines 0,2 0,5 0,3 mean that only one of the three arcs involved in the task will be implemented on the basis of a random choice with specified probabilities (0,3, 0,6, 0,1) (the logical stochastic exclusive "OR" operation).

The lines mean that only one of the three arcs involved in the task will be implemented on the basis of a deterministic choice of a project by the manager (the logical deterministic exclusive "OR" operation).

Let us note that the SATM can be reduced to classical models of the types GERT [5], CAAN [4], GAAN [2, 3], etc. For this, it is necessary to remove all the reverse arcs with negative parameters. The information on these arcs, the nodes that bind them together, and the parameters of these arcs is put in a special data file Q and, hence, "stored."

In the case of the inverse transition to the SATM from other networks, the information on the arcs whose boundary nodes enter into the model subject to the transformation (for example, the GAAN model) is chosen out of the filled file Q of reverse arcs. The obtained reverse arcs are inserted into the model that is thus brought to the standard form of the SATM.

As noted above, the choice of one of the directions at deterministic points of branching means the decision-making in implementing a project. For such a decision to be optimal in nature, it is necessary to solve on the SATM an optimization problem, which involves considerable difficulties. In order to solve the problem, we suggest transforming SATMs into GAANs [2, 3], for which similar optimization problems are amenable to an exact solution.

In conclusion, we will note that hereafter we will use the notion of consistency for SATMs, i.e., for networks with probabilistic parameters and alternatives. We will call an SATM inconsistent if at least one inconsistent realization of the SATM can appear with probability different from zero. It is easy to verify that any realization of the SATM is the GNM described above.

3. DESCRIPTION OF THE GAAN

In describing the GAAN, we will adhere to the classical terminology of network models, adopted in the earlier developed models of the types CPM, PERT, and GERT [6]. The GAAN model is a finite oriented and acyclic network graph $G(N, A)$ in which tasks are represented by graph arcs $(i, j) \in A$, and events by nodes (vertices) $i \in N$. The graph $G(N, A)$ has one initial node and no less than two terminal nodes. Each arc-task $(i, j) \in A$ relates to one of the three different kinds of arcs (tasks).

I. An arc (i, j) has a logical "AN" operation at the exit from the node i and at the entry into the node j.

II. An arc (i, j) accomplishes at the exit from the node i a logical "exclusive OR" operation of the probabilistic type. The probability $0 < p_{ij} < 1$ correlates to each arc $(i, j) \in A$, in which case no less than two alternative arcs emerge from the node i.

III. An arc (i, j) implements at the exit from the node i a logical "exclusive OR" operation of the deterministic type. At the node i (known as the decision-making node), the manager of the project makes the decision about the choice of one of the alternative arcs.

The arcs of all three types can go out of one and the same node $i \in N$ (except for the terminal one). The arcs of all three types can enter into any node $i \in N$ (except for the initial one). We will define an admissible version of the GAAN model. Hereafter, by an admissible version we will refer to a subgraph $G^*(N^*, A^*)$ that satisfies the following conditions:

(a) the subgraph $G^*(N^*, A^*)$ has one initial node;

(b) if the node i belongs to N^*, then the subgraph $G^*(N^*, A^*)$ includes all alternative arcs of the first and second types that come out from this node;

(c) if the node i belongs to N^* and this node was the source of the collection of alternative arcs of the third type in the initial network $G(N, A)$, then the admissible version $G^*(N^*, A^*)$ includes only one alternative arc in this collection;

(d) the subgraph $G^*(N^*, A^*)$ is the maximum subgraph that satisfies conditions (a) and (b).

A set of algorithms performing an exhaustive search for all admissible versions of the GAAN model for obtaining an optimal solution are described in [2, 3]. Let us note that any admissible version $G^*(N, A)$ is a network model of the GERT type [5].

4. SOLUTION OF THE OPTIMAL PROBLEM FOR THE CHOICE OF AN ADMISSIBLE VERSION

We will state the problem for the choice of an optimal admissible version $G^{*\mathrm{opt}}(N, A)$ out of R admissible versions $G^*_r(N, A)$, $1 \leqslant r \leqslant R$:

$$K_1\left\{E\left[G^{*\mathrm{opt}}(N, A)\right]\right\} = \operatorname*{Opt}_{1 \leqslant r \leqslant R}\left\{K_1\left\{E\left[G^*_r(N, A)\right]\right\}\right. \quad (1)$$

under the constraints

$$K_V\left\{E\left[G^{*\mathrm{opt}}(N,A)\right]\right\} \leqslant H_V, \quad 2 \leqslant v \leqslant n, \tag{2}$$

where H_V identifies prescribed constraints on $(n-1)$ criteria $K_2, \ldots, K_n$ and E is the symbol of a mean value (expectation).

For the criterion K, we can adopt, for example, the function of minimization of the time it takes to implement a project, while for O_2 we can accept the expectation of the cost of carrying out the project. It is also possible to use other criteria, for example, the reliability, entropy [2], resource criteria, etc.

It is shown in [2] that the problem (1)–(2) is an **NP**-complete one. Consequently, it can be solved only by the exhaustive method.

The idea of renumbering the admissible versions for one of the earlier developed CAAN models [4] reduces to the use of the lexicographic method of path-tracing of the set of maximum paths. For the GAAN model, the ordering on the set of paths is replaced by the ordering on the set of subnetworks [2]. In this case, there is a need to introduce the one-to-one correspondence between the set of admissible versions and the set of vectors with the number of coordinates equal to the number of interior points of the initial graph GAAN, namely, the graph $G(N,A)$. For any finite set of these vectors, the introduction of the ordering by means of the lexicographic method is no longer a specific problem. Thus, we first construct an admissible version that corresponds to the vector of the minimum ordinal number. Further, we construct the successive lexicographically ordered vector and subsequently define an admissible version, and so on, until we obtain the admissible version of the maximum ordinal number. The renumbering algorithm is easy to use, and it is readily programmed on a computer.

5. QUESTIONS OF OPTIMAL CONTROL ON THE BASIS OF ADMISSIBLE VERSIONS

We described above the idea of renumbering, which makes it possible to perform an exhaustive search for all admissible versions $G^*(N,A)$. It can be shown [2] that in this case, none of the admissible versions will be left out. We assume that in the course of the solution of the optimal problem (1)–(2), we defined an optimal admissible version $G^{*\mathrm{opt}}(N,A)$. This version, like all the remaining versions, is one of the versions of the development of a project on the basis of the decision made by the management. Let us note that the decision-making in the process of control of network models with multivariant outcomes implies the choice of a project at each node, which will be reached in the course of the project implementation and from which deterministic alternative tasks of only one of the possible directions emerge. Therefore, the problem of optimal control of the project is, in principle, equivalent to the problem (1)–(2) of the choice of an optimal admissible version. If we adopt a nonrealistic assumption that a network model takes on a static character and the model parameters do not change during the implementation of the project, then this means that the optimal admissible version determines the entire course of the project development from start to finish. However, the network information actually undergoes changes in the course of implementation of the project. From this fact it transpires that the optimal admissible version also can experience changes during the implementation of the project. Relying on the above discussion, we suggest the following technique of optimal control.

Step 1. At each node with alternative deterministic outcomes for the SATM project, the node being made active at the instant t in the course of development of the project, it is necessary to accomplish the following:

(a) perform the transformation of the network model $N_t(A,L)$ (more exactly, the subnetwork unrealized by the instant t) into the GAAN model, which we denote by $G_t(N,A)$;

(b) solve the problem (1)–(2) for the $G_t(N,A)$ model, which enables us to construct all admissible versions and select from them an optimal version $G_t^{*\mathrm{opt}}(N,A)$;

(c) perform the inverse transformation of the $G_t^{*\mathrm{opt}}(N,A)$ subnetwork into the $N_t^*(A,L)$ network of the SATM;

(d) check the obtained $N_t^*(A,L)$ model for consistency; if the $N_t^*(A,L)$ network is inconsistent, it is necessary to pass on again to the set of admissible versions $\{G_t^*(N,A)\}$, exclude from consideration the version $G_t^{*\mathrm{opt}}(N,A)$, and solve again the optimal problem (1)–(2).

Next, it is necessary to carry out again the operations at stages (b) and (c) until the $N_t^*(A,L)$ network becomes consistent. Let us note that if we perform the inverse transformation of all admissible versions $G_t^*(N,A)$ into the $N_t^*(A,L)$ subnetworks that conform to these versions, then the optimal admissible version $G_t^{*\mathrm{opt}}(N,A)$ may not correspond to the optimal version $N_t^{*\mathrm{opt}}(A,L)$ obtained by the exhaustive search for requisite $N_t^*(A,L)$ networks. Therefore, we have the right to consider the technique involved as being only quasioptimal, the more so as we take into account the check for consistency at the stage (d).

Step 2. Implement a version of the SATM project in accordance with the direction selected in the $G_t^{*\text{opt}}(N, A)$ network at the branching point. The case at hand is the choice of the direction for the $N_t^*(A, L)$ network obtained at stage (d) of Step 1.

After the choice of the optimal direction, we need to continue to work out the project up to the next decision point (with deterministic branching) at a new instant of time t. For the remaining part of the $N_{t'}^*(A, L)$ project, we have to introduce all the changes in the values of the parameters that took place in the time period $[t, t']$. Thereafter, we need to pass to Step 1, i.e., to solve again the problem (1)–(2) for the choice of a new optimal direction. Let us note that if no changes were introduced in the project over the period $[t, t']$, then the choice of the optimal direction from the subsequent branching points could be made in accordance with the admissible version $G_t^{*\text{opt}}(N, A)$ obtained from the solution of the problem (1)–(2) at the instant of time t.

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