Automation and Remote Control, Vol. 63, No. 8, 2002, pp. 1329–1342. Translated from Avtomatika i Telemekhanika, No. 8, 2002, pp. 136–150.

CONTROL IN SOCIAL ECONOMIC SYSTEMS

A Model of Optimal Control of Structural Changes in an Organizational System

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Received March 18, 2002

Abstract—An organizational system is considered, namely, an aggregate of certain elements that are hierarchically connected for fulfilling a collection of functions. It is assumed that the result of the work of the system depends on its structure. For a set of various structures (graphs of the organization), expenditures on the operation of the system and on the restructurization are defined. An optimal control of structural changes in a finite time interval involves the choice of the sequence of structural transformations that maximize the total profit. An optimal control enables balancing the expenditures for the operation (effectiveness) and for the restructurization (the stability to external actions).

1. INTRODUCTION

The great majority of the available models of an organizational system presupposes the structure of a specified system or examines a few versions of it. The two-level organizational system of the general form (the center and subordinate elements) is thoroughly studied in the theory of active systems. One of the approaches to the study of a multilevel system involves its decomposition into a number of two-level systems, which affords studying the system with the invariable structure. However, the retention of the effectiveness of the system under changes of the external medium (environment) sometimes requires its structural reconstruction, which is not described in the framework of two-level systems [1]. In the subsequent discussion, we examine a model that enables us to compare the effectiveness of various controls of structural changes.

We will consider an organizational system over the course of T units of time. The parameters corresponding to a unit time t will be provided with the upper index t, $t = \overline{1,T}$.

We assume that the system receives a profit as it turns out some products out of the prescribed collection $I_1, \dots, I_q$ defining the field of activities (the branch). The composition of output products can change with time, but the entire collection $I_1, \dots, I_q$ of products of the branch will be considered to be invariable in the entire time interval under study.

We will denote the volume of products I_k put out in the unit time t by $y_k^t \geq 0$, and the vector of volumes by $\mathbf{y}^t = (y_1^t, \dots, y_q^t)$. The price of a product I_k for the unit time will be denoted by p_k^t and a maximum volume of products that the system can sell at the market will be denoted by v_k^t. The pertinent vectors will be designated as $\mathbf{p}^t = (p_1^t, \dots, p_q^t)$, $\mathbf{v}^t = (v_1^t, \dots, v_q^t)$.

We consider that the parameters $\mathbf{p}^1, \dots, \mathbf{p}^T$ and $\mathbf{v}^t, \dots, \mathbf{v}^T$ are defined by the external medium and are independent of the control of the system. The collections of vectors $\mathbf{p}^t = \{\mathbf{p}^1, \dots, \mathbf{p}^t\}$ and $\mathbf{v}^t = \{\mathbf{v}^1, \dots, \mathbf{v}^t\}$ are thought to be known by the unit time t. The parameters $\mathbf{y}^1, \dots, \mathbf{y}^T$ are controllable: they can be chosen with due regard for constraints $0 \leq y_k^t \leq v_k^t$, $k = \overline{1,q}$, $t = \overline{1,T}$.

The array of operations (elementary jobs) necessary for the output of all products will be designated as $e_1, \dots, e_r$. We assume that the array $e_1, \dots, e_r$ does not contain auxiliary operations involved with the organization of the system (control, accounting, etc.) and depends on the technology.

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Let us prescribe the matrix $W = \{w_{k,j}\}$ of technological coefficients, where $w_{k,j} \geq 0$ is the number of units of an elementary job e_j required for the output of the unit of a product I_k, $k = \overline{1,q}$, $j = \overline{1,r}$. We assume that the array $e_1, \dots, e_r$ and the matrix W are identical for all organizations of the given branch and invariable parameters of the external medium in the entire time interval under examination.

For a length of the unit time t, the system has available a certain set of performers $A^t = \{a_1^t, \dots, a_{n^t}^t\}$, who can do elementary jobs $e_1, \dots, e_r$. By the next unit time, there is a possibility of dismissing some performers from A^t and engage performers from the set $\widetilde{A}^t$. Thus, by controlling the system, it is possible to select the set $A^t \subseteq A^{t-1} \cup \widetilde{A}^{t-1}$, $t = \overline{1,T}$, where A^0 is the initial set of performers and $\widetilde{A}^0 = \varnothing$. The sets A^0, $\widetilde{A}^1, \dots, \widetilde{A}^T$ depend on the external medium and are independent of the control. By the unit time t, the array of sets $\widetilde{\mathbf{A}}^t = \left\{\widetilde{A}^1, \dots, \widetilde{A}^{t-1}\right\}$ is known, i.e., the information on the labor market in the past.

We will denote by $s_j(a)$ the amount of an elementary job e_j that the performer can carry out in the unit time. The vector $\mathbf{s}(a) = (s_1(a), \dots, s_r(a))$ of the productivity of the performer is taken to be constant in the entire time interval under consideration (disregarding changes of $\mathbf{s}(a)$, for example, in the training or degradation).

Let us denote by $0 \leq x_j^t(a) \leq 1$ a share of the unit time t that the performer a spends to accomplish the elementary job e_j. The work intensity of the performer a will be designated as $z^t(a) = \sum_{j=\overline{1,r}} x_j^t(a)$. The plan of jobs of the performer a for the unit time t will be designated as $\mathbf{x}^t(a) = (x_1^t(a), \dots, x_r^t(a))$. By controlling the system, it is possible to select $\mathbf{x}^t(a)$ with due regard for the constraint $z^t(a) \leq 1$. To perform the amount of jobs that is necessary to turn out products, the following relations must be met: $\sum_{k=\overline{1,q}} y_k^t w_{k,j} \leq \sum_{a \in A^t} s_j(a) x_j^t(a)$, $j = \overline{1,r}$. If all inequalities are met, then the plan $\mathbf{y}^t$ of the output and the plans $\mathbf{x}^t(a)$ of jobs, $a \in A^t$, will be called correct.

The expenditures for the accomplishment of jobs by the performer a in accordance with the plan $\mathbf{x}^t(a)$ result from the technology (necessary materials, energy, etc.) and are independent of the interaction of performers. We assume that the control does not affect the wage (the fixed or the piece one that depends on $\mathbf{x}^t(a)$). Thus, the total expenditures $p(a, \mathbf{x}^t(a))$ for the upkeep of the performer a depend on the performer himself and on the plan of his jobs. It the plans $\mathbf{x}^t(a)$ of jobs are defined, then the expenditures will be designated as $p^t(a) = p(a, \mathbf{x}^t(a))$.

2. THE GRAPH OF ORGANIZATION OF THE SYSTEM

To turn out products, it is necessary to organize the interaction of performers, each of which performs a share of elementary jobs.

Definition 1. Any nonempty subset $f \subseteq A^t$, $t = \overline{1,T}$, will be called a group. The set of all groups for the unit time t will be denoted by $F^t = 2^{A^t} \setminus \{\varnothing\}$. The cardinality of the group f will be called the number of performers $|f|$ contained in it.

The distribution of jobs $x_1^t(a)s_1(a), \dots, x_r^t(a)s_r(a)$ completed by the performer a for the output of some or other products determines his participation in the output of each of the products. Thus, a certain subset $f \subseteq A^t$ of performers takes part in the output of products I_k, in which case $f = \varnothing$ at $y_k^t = 0$, otherwise $f \in F^t$.

Consequently, to turn out products in volumes $\mathbf{y}^t$ for the unit time t, it is necessary to organize the array of groups $\mathbf{f}^t = \{f_1^t, \dots, f_{m^t}^t\}$, $m^t \leq q$. According to the plans $\mathbf{y}^t$ of the output and the plans $\mathbf{x}^t(a)$ of jobs, generally speaking, the array $\mathbf{f}^t$ is defined ambiguously. We will call the array correct if it corresponds to a certain distribution of jobs accomplished by performers over products.

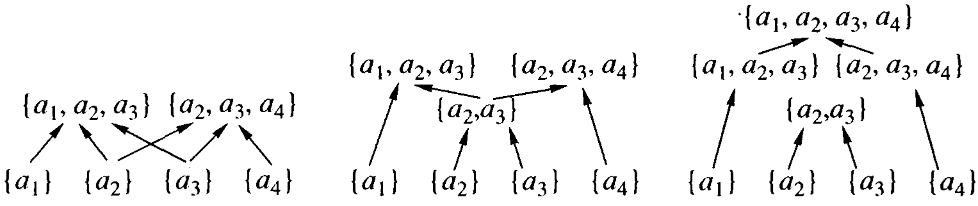

Fig. 1. Examples of organization graphs.

The control of the system determines one of the correct arrays of groups. On fixing the unit time, we will omit the index t later on in this item and the subsequent ones.

Definition 2. We will call the oriented graph $G = (V, E)$ the graph of the organization of groups $f_1, \dots, f_m$ if it satisfies the following conditions:

(a) the vertices correspond to the groups, i.e., $V \subseteq F$, $f_1, \dots, f_m \in V$;

(b) $E \subseteq V \times V$ and $g \subset h$ is fulfilled for any edge $(g, h) \in E$;

(c) for an arbitrary vertex $g \in V$, we denote by $Q(g) = \{h : (h, g) \in E\}$ the set of vertices from which edges extend into g. Then $h' \not\subset \bigcup_{h \in Q(g) \setminus \{h'\}} h$ is fulfilled for any $h' \in Q(g)$. For any $g \neq \{a_i\}$, $i = \overline{1, n}$, we have $g = \bigcup_{h \in Q(g)} h$, $Q(\{a_i\}) = \varnothing$.

Thus, the edges entering into the group $g \in V \setminus \{a_1\}, \dots, \{a_n\}$ determine the array of subgroups $Q(g)$ that form this group. Each subgroup $Q(g)$ is not covered entirely by the remaining subgroups (otherwise, there is no point in using it for the formation of g). Edges do not enter into groups $\{a_1\}, \dots, \{a_n\}$ (there is no need to organize groups consisting of one performer). The graph defines the order of the interaction of performers in organizing the groups $f_1, \dots, f_m$, i.e., the structure of the system. Under organization will be understood the appropriate graph of the organization. Vertices (groups) of a cardinality of 1 will be called elementary and the nonelementary vertices of G that are different from $f_1, \dots, f_m$ will be called intermediate.

From Definition 2 it follows that the organization graph is acyclic and $|Q(g)| \geq 2$ is fulfilled for any $g \in V \setminus \{a_1\}, \dots, \{a_n\}$.

Definition 3. The organization $G = (V, E)$ will be called sequential if for any nonelementary group $g \in V$, we have $Q(g) = \{g \setminus \{a\}, \{a\}\}$ for some $a \in A$.

Definition 4. The organization $G = (V, E)$ will be called the r-organization, $r \geq 2$, if $|Q(g)| \leq r$ for any group $g \in V$.

Definition 5. The organization $G = (V, E)$ will be called simultaneous if $V = \{\{a_1\}, \dots, \{a_n\}, f_1, \dots, f_m\}$, in which case $Q(f_i) \subseteq \{\{a_1\}, \dots, \{a_n\}\}$ for $i = \overline{1, m}$.

In the sequential organization, any nonelementary group is set up of two subgroups, although at least one of them is elementary. Thus, the sequential organization is a particular case of the 2-organization. The simultaneous organization is unique. Examples of organizations are given in Fig. 1.

At the left of Fig. 1, the simultaneous organization of the groups $f_1 = \{a_1, a_2, a_3\}$ and $f_2 = \{a_2, a_3, a_4\}$ is shown, in which performers interact between themselves in the groups f_1 and f_2 without intermediate links. An example of the sequential organization of the groups f_1 and f_2 is given at the center. The performers a_2 and a_3 interact with each other, forming an intermediate group $\{a_2, a_3\}$ (the subdivision of the organization), which is used both for the organization of f_1 and for the organization of f_2. At the right of the figure, an example of the 2-organization of the group $f = \{a_1, \ldots, a_4\}$ is shown. Here, f is formed from two intersecting intermediate subgroups.

Definition 6. The terminal vertex of the graph of the organization will be called the vertex from which edges do not emerge.

Definition 7. An organization will be called an organization without intersections if the set $Q(g)$ does not contain intersecting groups for all nonelementary groups $g \in V$.

Definition 8. The vertex $g \in V$ will be called the daughter node for the vertex $f \in V$, $f \neq g$, if in the graph $G = (V, E)$ there exists a path from g to f.

The daughter node (group) is a certain subset of the parent node. We will prove an auxiliary assertion that will be used later on.

Assertion 1. *The organization of one group f without intersections, which contains the single terminal vertex f, represents a tree with the root in f.*

The proof is carried out by induction over the cardinality of f. If $|f| = 1$, then the organization $G = (V, E)$ consists of one vertex because, otherwise, there would exist a terminal vertex different from f. Let the assertion be proved for all cardinal numbers that are less than k.

We assume that $f = \{a_1, \ldots, a_k\}$ is formed from subgraphs $Q(f) = \{g_1, \ldots, g_\ell\}$, $g_i \cap g_j = \varnothing$, for $i \neq j$. Let G_i be a subgraph of G, which consists of g_i and its daughter nodes (subsets g_i). Any vertex $h \neq f$ belongs to one of the subgraphs G_i, $i = \overline{1, \ell}$, because f is the single terminal vertex. If $g \in G_i$ and $h \in G_j$, $i \neq j$, then $(g, h) \notin E$, otherwise $g \subset g_i \cap g_j = \varnothing$.

By supposition, G_i is a tree with the root in g_i. Consequently, G consists of f into which edges pass from the roots ℓ of independent trees, i.e., G is the tree with the root in f.

3. COST OF ORGANIZATION OF THE SYSTEM. OPTIMAL ORGANIZATION

By Definition 2, edges from $Q(g) = \{g_1, \ldots, g_k\}$, $k \geq 2$, $g = g_1 \cup \ldots \cup g_k$ enter into the nonelementary vertex g of the graph of the organization. An arbitrary array of the subgroups satisfying the above conditions will be called admissible. The joint work of subgroups $g_1, \ldots, g_k$ in the group g requires expenditures on the coordination of actions of the subgroups in the group (control), the registration of results, and other overhead charges.

Let us consider an admissible array $g_1, \ldots, g_k$. We assume that the cost of the organization of the joint work of the subgroups $g_1, \ldots, g_k \in F$ in the group $g = g_1 \cup \ldots \cup g_k$ for the unit time is defined by the functional of the organization cost $P(g_1, \ldots, g_k) \geq 0$, which is preset on all admissible arrays of the subgroups and does not change in the rearrangement of arguments.

To an arbitrary vertex $g \in V \backslash \{a_1\}, \ldots, \{a_n\}$ of the organization graph $G = (V, E)$, we assign the cost $R(g) = P(g_1, \ldots, g_k)$ of its organization from the subgroups $Q(g)$, where $\{g_1, \ldots, g_k\} = Q(g)$. The cost functional is defined on this array of subgroups in view of its admissibility.

Definition 9. The cost of operation of the organization G will be called the quantity $P(G) = \sum_{g \in V \backslash \{a_1\}, \ldots, \{a_n\}} R(g) + \sum_{a \in A} p(a)$. The organization G^* of groups $f_1, \ldots, f_m$ will be called optimal if $P(G^*) = \min P(G)$, where the minimum is taken over all possible organizations of the groups $f_1, \ldots, f_m$.

Definition 10. The problem for an optimal organization will be called the problem of the search for one of the optimal organizations.

The cost of operation (or simply the cost) of an organization represents a sum of the expenditures for the organization of the joint work of performers in groups and the expenditures for the pay of performers (the latter expenditures do not depend on G and increase the cost of all organizations by a constant). An optimal organization minimizes the cost of operation of the system.

Definition 11. The cost functional will be called monotonic if the following conditions are met for any admissible array of subgroups $\{g_1, \ldots, g_k\}$:

(a) $P(g_1, \ldots, g_k) \leq P(g_1, \ldots, g_k, g)$, where g is an arbitrary subgroup at which the array $\{g_1, \ldots, g_k, g\}$ is admissible.

(b) $P(g_1, \ldots, g_k) \leq P(g_1, \ldots, g_{i-1}, g_{i+1}, \ldots, g_k)$ for any $i = \overline{1, k}$ and a subgroup g such that $g_i \subset g$ and the array $\{g_1, \ldots, g_{i-1}, g, g_{i+1}, \ldots, g_k\}$ is admissible.

Thus, in the case of the monotonic functional, the addition of one more subgroup or the extension of one of the subgroups does not lead to a decrease in the cost of the organization.

Theorem 1. *In the case of the monotonic cost functional, an optimal organization of one group f exists in the class of trees with the root in f.*

Proof. Let $G = (V, E)$ be an optimal organization of the group. We will remove terminal vertices different from f, in which case the optimality will remain. We will construct an optimal organization G^* of the group, in which exactly one edge emerges from each vertex $g \in V \setminus \{f\}$.

Let $g \in V$ be the vertex of the highest cardinality, from which at least two edges emerge: one into h_1 and the other into ℓ_1. From h_1 and ℓ_1 there exist paths to f (there are no other terminal vertices), i.e., two paths exist from g to f. We will denote their sections up to the first intersection by $g - h_1 - h_2 - \ldots - h_{n_1}$ and $g - \ell_1 - \ell_2 - \ldots - \ell_{n_2}$, where $h_{n_1} = \ell_{n_2}$. The groups $h_1, \ldots, h_{n_1-1}$, $\ell_1, \ldots, \ell_{n_2-1}$ have a higher cardinality than g and, hence (by definition of g) exactly one edge goes out of them to the next vertex of the path.

For any edge $(\ell, h) \in E$, the following reconstruction will be called the (ℓ, h)-simplification. We will use the notation $h' = \bigcup_{h'' \in Q(h) \setminus \{\ell\}} h''$. If $h' \in V$ (in particular, at $|Q(h)| = 2$), then we will remove h and incoming edges. If $h' \notin V$, we will take away the edge (ℓ, h). In this case, h will change to h' (the group h' can be set up of the array $Q(h) \setminus \{\ell\}$ in view of its admissibility). We will call the vertex h' the result of the (ℓ, h)-simplification. It is obvious that $h \setminus h' \subseteq \ell$. In view of the monotonicity of the functional, the (ℓ, h)-simplification does not increase the cost of the graph.

We will perform the (g, ℓ_1)-simplification and denote the result by ℓ'_1. The cost of the graph did not increase. We have $\ell_1 \setminus \ell'_1 \subseteq g$. The edge (ℓ_1, ℓ_2) emerged from ℓ_1. We introduce the designation $\ell'_2 = \bigcup_{h \in (Q(\ell_2) \setminus \{\ell_1\}) \cup \{\ell'_1\}} h$. If $\ell'_2 = \ell_2$, we obtained the organization G' of the group f. Otherwise, we continue the reconstruction.

If $\ell'_2 \in V$, then we remove ℓ_2 and the edges entering into it. But if $\ell'_1 \subseteq \bigcup_{h \in Q(\ell_2) \setminus \{\ell_1\}} h$, we carry out the (ℓ_1, ℓ_2)-simplification; otherwise, ℓ'_2 is formed from the admissible array $(Q(\ell_2) \setminus \{\ell_1\}) \cup \{\ell'_1\}$. As a result, instead of ℓ_2, in all cases ℓ'_2 will be arranged. Also, on account of the monotonicity of the functional, the cost of the graph does not increase. We have $\ell_2 \setminus \ell'_2 \subseteq \ell_1 \setminus \ell'_1 \subseteq g$.

If $\ell'_2 \neq \ell_2$, then in a similar way (accurate to the replacement of ℓ_1 by ℓ_2 and ℓ_2 by ℓ_3), we rearrange the graph without an increase in the cost. Instead of ℓ_3, we will organize ℓ'_3, $\ell_3 \setminus \ell'_3 \subseteq g$, and so on. If ℓ_{n_2-1} is replaced by $\ell'_{n_2-1} \neq \ell_{n_2-1}$, then in view of $\ell_{n_2-1} \setminus \ell'_{n_2-1} \subseteq g$ and $g \subseteq h_{n_1-1} \in$

$Q(\ell_{n_2}) \setminus \{\ell_{n_2-1}\}$, we have $\ell'_{n_2} = \ell_{n_2}$, i.e., at a certain step, we will obtain the organization G' of the group f, $P(G') \leq P(G)$.

We remove terminal vertices of G' that are different from f. If g is not removed and more than one edge emerges from it, then we carry out a similar operation with the graph G' instead of G. As a result, either we will remove g or exactly one edge will go out of g.

If in the obtained graph there are vertices from which more than one edge emerges, we will perform the actions described above. In the course of reconstructions, edges are not added. As a result, we obtain the desired organization $G^* = (V^*, E^*)$. From each elementary group $\{a\} \subseteq f$ in G^* there exists exactly one path to f. For any nonelementary $g \in V^*$ and any g_1, $g_2 \in Q(g)$, the relation $g_1 \cap g_2 = \varnothing$ is fulfilled, otherwise two paths to f from the group $\{a\} \subseteq g_1 \cap g_2$ would exist. Thus, G^* is the optimal organization without intersections and terminal vertices different from f. From Assertion 1 it follows that G^* is a tree with the root in f.

Definition 12. The functional P will be called convex if for any admissible array of groups $\{f_1, \dots, f_k\}$, $k \geq 3$, there exists a subarray $\{g_1, \dots, g_r\} \subset \{f_1, \dots, f_k\}$, $2 \leq r < k$, for which the following inequality is met:

$$P(f_1, \dots, f_k) \geq P(g_1, \dots, g_r) + P(g, h_1, \dots, h_{k-r}), \tag{a}$$

where $g = g_1 \cup \dots \cup g_r$, $\{h_1, \dots, h_{k-r}\} = \{f_1, \dots, f_k\} \setminus \{g_1, \dots, g_r\}$. The functional P will be called concave if there does not exist a subarray $\{g_1, \dots, g_r\}$, for which the inequality (a) is strictly fulfilled, i.e., for any subarray $\{g_1, \dots, g_r\} \subset \{f_1, \dots, f_k\}$, $2 \leq r < k$, the following inequality is met:

$$P(f_1, \dots, f_k) \leq P(g_1, \dots, g_r) + P(g, h_1, \dots, h_{k-r}). \tag{b}$$

In the case of the convex functional, instead of the combining of subgroups $f_1, \dots, f_k$ into the group $f = f_1 \cup \dots \cup f_k$, it is possible, without increasing the cost, to organize initially some subgraphs from $f_1, \dots, f_k$ and then to combine the obtained group with the remaining subgroups from $f_1, \dots, f_k$. In the case of the concave functional, if is impossible to reduce the cost in this way.

Definition 13. Let us prescribe a certain set of admissible arrays of groups. If for any array of the set, the inequality (a) of Definition 12 is met, then the functional P will be called convex on the given set, and if the inequality (b) is met, the functional will be called concave.

Theorem 2. *In the case of the convex functional of the cost, an optimal organization exists in the class of 2-organizations.*

Proof. We will consider the optimal organization $G = (V, E)$. Let $k = \max |Q(g)|$, where the maximum is taken over all nonelementary vertices of G. If $k = 2$, then an optimal 2-organization is found.

Let $k \geq 3$. We will find $f \in V$ for which $Q(f) = \{f_1, \dots, f_k\}$. In view of the convexity of the functional, there exists a subarray $\{g_1, \dots, g_r\} \subset \{f_1, \dots, f_k\}$, $2 \leq r < k$, for which $P(f_1, \dots, f_k) \geq P(g_1, \dots, g_r) + P(g, h_1, \dots, h_{k-r})$, where $\{h_1, \dots, h_{k-r}\} = \{f_1, \dots, f_k\} \setminus \{g_1, \dots, g_r\}$, $g = g_1 \cup \dots \cup g_r$. We will rearrange G. If $g \notin V$, then we will supplement g, forming it from $g_1, \dots, g_r$. We will change edges entering into f so that $Q(f) = \{g, h_1, \dots, h_{k-r}\}$. In view of the admissibility of the arrays $\{g_1, \dots, g_r\}$ and $\{g, h_1, \dots, h_{k-r}\}$, the reconstruction is possible (it is shown in Fig. 2 at $\{g_1, \dots, g_r\} = \{f_1, \dots, f_r\}$). The obtained graph G' is the graph of the organization of the same groups as those of G, in which case $P(G') \leq P(G)$.

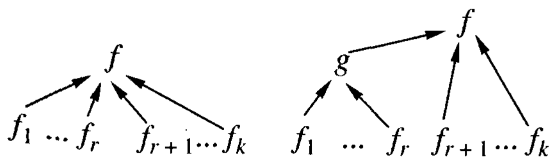

Fig. 2. Reconstruction of an optimal graph involved with a convex functional.

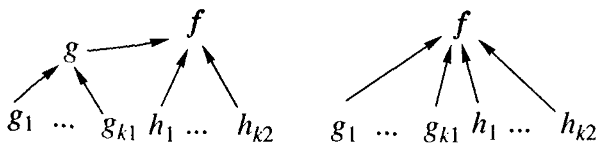

Fig. 3. Reconstruction of an optimal tree involved with a concave functional.

We have $|Q'(f)| = k - r + 1 < k$, $|Q'(g)| = r < k$, i.e., no less than k edges enter into f and g. The number of the vertices into which k edges enter decreased by unity. Performing similar reconstructions, we will ultimately obtain an optimal organization for which $k_1 = \max|Q(g)| < k$. If $k_1 > 2$, then we will repeat the discussion. As a result, we will arrive at an optimal 2-organization.

Corollary. *In the case of the monotonic cost functional that is convex on the arrays of nonintersecting groups, an optimal organization of one group f exists in the class of 2-trees with the root in f.*

Proof. By Theorem 1, there exists an optimal tree of the organization with the root in f. We will take it as the graph G in proving Theorem 2. For any vertex $f \in V$, the array $Q(f) = \{f_1, \dots, f_k\}$ does not contain intersecting groups. Consequently, to reconstruct G, it is sufficient to have the convexity on the arrays of nonintersecting groups. After the reconstruction, we obtain a tree with the root in f, which enables us to continue the discussion.

Theorem 3. *In the case of the monotonic cost functional that is concave on the arrays of nonintersecting groups, the simultaneous organization of one group is optimal.*

Proof. By Theorem 1, there exists an optimal tree $G = (V, E)$ of the organization of one group f. If G does not contain intermediate vertices, than G is a simultaneous organization. Otherwise, we will consider the intermediate vertex $g \in V$ of the highest cardinality. Let $Q(g) = \{g_1, \dots, g_{k_1}\}$. Exactly one edge passes from g into f. Let $Q(f) = \{g, h_1, \dots, h_{k_2}\}$. In the array $Q'(f) = \{g_1, \dots, g_{k_1}, h_1, \dots, h_{k_2}\}$, there is no intersecting groups (G is a tree), i.e., $Q'(f)$ is admissible and the functional is concave on it: $P(g_1, \dots, g_{k_1}, h_1, \dots, h_{k_2}) \le P(g_1, \dots, g_{k_1}) + P(g, h_1, \dots, h_{k_2})$. We will remove g and organize f from the array $Q'(f)$ (Fig. 3).

We will obtain an optimal tree of the organization of the group f that contains one intermediate vertex less than G. Continuing these actions, we will prove the optimality of the simultaneous organization of one group.

Definition 14. The cost functional will be called substantially convex if it is convex and for any admissible array of nonelementary groups $\{g_1, g_2\}$, at least one of the two conditions is fulfilled:

(a) for any $a \in g_1$: $P(g_1, g_2) \ge P(g_1 \setminus \{a\}, g_2) + P((g_1 \setminus \{a\}) \cup g_2, \{a\})$;

(b) for any $a \in g_2$: $P(g_1, g_2) \ge P(g_1, g_2 \setminus \{a\}) + P(g_1 \cup (g_2 \setminus \{a\}), \{a\})$,

in which case the functional is taken to be equal to zero if the associated array is inadmissible.

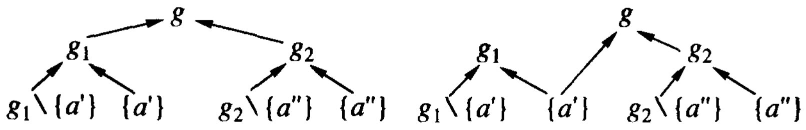

Fig. 4. First version of reconstruction involved with a substantially convex functional.

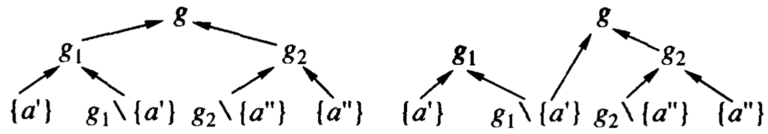

Fig. 5. Second version of reconstruction involved with a substantially convex functional.

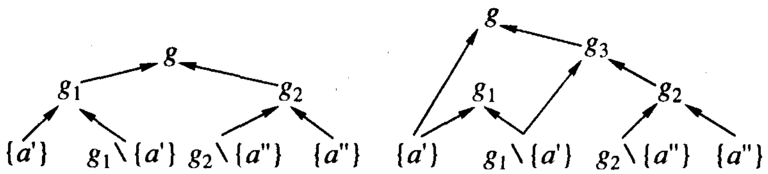

Fig. 6. Third version of reconstruction involves with a substantially convex functional.

In other words, the functional is substantially convex if in the organization of two subgroups, it is possible to remove an arbitrary performer from one subgroup and then to organize it with the obtained group without increasing the cost.

Theorem 4. *In the case of a substantially convex cost functional, an optimal organization exists in the class of sequential organizations.*

Proof. Let $G = (V, E)$ be an optimal 2-organization that exists in view of Theorem 2. We will call a vertex of the graph incorrect if it is arranged from two nonelementary groups. Otherwise, we will call it correct. If the graph G does not contain incorrect vertices, then G is the sequential organization. Otherwise, we will produce an optimal 2-organization G^* in which the number of incorrect vertices is less than in G by one. Thus, performing this operation the required number of times, we will arrive at an optimal sequential organization.

Let g be an incorrect vertex such that all daughter nodes of which are correct. Let $Q(g) = \{g_1, g_2\}$. Then, g_1 and g_2 are nonelementary correct vertices and, hence, $Q(g_1) = \{g_1 \setminus \{a'\}, \{a'\}\}$ and $Q(g_2) = \{g_2 \setminus \{a''\}, \{a''\}\}$. The functional is substantially convex. Let the condition (a) of Definition 14 be met.

If the array $\{g_1 \setminus \{a'\}, g_2\}$ is inadmissible, then $(g_1 \setminus \{a'\}) \subset g_2$ and, hence, $g_2 \cup \{a'\} = g$, $\{a'\} \not\subset g_2$. We will arrange g from g_2 and $\{a'\}$ (Fig. 4). If $(g_1 \setminus \{a'\}) \cup g_2 = g_2$ is met, the inequality (a) takes the form $P(g_1, g_2) \geq P(g_2, \{a'\})$. We obtain the sequential organization G', $P(G') \leq P(G)$.

If the array $\{g_1 \setminus \{a'\}, g_2\}$ is admissible, then we add the vertex $g_3 = (g_1 \setminus \{a'\}) \cup g_2$, forming it from $g_1 \setminus \{a'\}$ and g_2. In this case, if $\{a'\} \subset g_3$, then $g_3 = g$. We obtain the sequential organization G' (Fig. 5). In view of the inadmissibility of $\{(g_1 \setminus \{a'\}) \cup g_2, \{a'\}\}$, the inequality (a) assumes the form $P(g_1, g_2) \geq P(g_1 \setminus \{a'\}, g_2)$. Consequently, $P(G') \leq P(G)$.

If $\{a'\} \not\subset g_3$, then we establish g from $\{a'\}$ and g_3. We produce the sequential organization G' (Fig. 6). In view of $P(g_1, g_2) \geq P(g_1 \setminus \{a'\}, g_2) + P(g_3, \{a\})$, we have $P(G') \leq P(G)$.

If the condition (b) of Definition 14 is met, then we reason in the same way, replacing g_1 by g_2 and $\{a'\}$ by $\{a''\}$. Thus, in all cases we obtained the optimal sequential organization G' in which g is correct. If G' does not contain g_3 or g_3 is correct, then G' has one incorrect vertex less than G, i.e., the desired organization G^* is established.

Let G' contain the incorrect vertex g_3. The cardinality of g_3 is less than that of g and all daughter nodes of g_3 are correct. We will repeat the reconstruction, taking the graph G' instead of G and the vertex g_3 instead of g, which will decrease again the cardinality of the incorrect vertex. Repeating these actions, we will obtain G^* at the next step or reach the instant at which the cardinality of g_3 is equal to two. In this case, g_3 is correct, i.e., the desired organization G^* is built up.

4. EXAMPLES OF COST FUNCTIONALS

Let us assume that the complexities $c_1^e \geq 0, \ldots, c_r^e \geq 0$ of units of the elementary job (for example, mean labor expenditures) are preset, which are dimensionless comparable indices.

Definition 15. We will define the complexity (potential) $C(a)$ of the performer a as a maximum of the complexity of the elementary job that the performer a is able to carry out in a unit time: $C(a) = \max(c_1^e s_1(a), \ldots, c_r^e s_r(a))$. The complexity (potential) of the group f will be given by the quantity $C(f) = \left(\sum_{a \in f} C(a)^{1/\alpha}\right)^\alpha$, where $\alpha \in (0, +\infty)$.

At $\alpha = 1$, the potential of the group is equal to a sum of the potentials of performers who comprise the group; for $\alpha > 1$, it is higher than this sum and for $\alpha < 1$, it is lower. The cost of the organization of subgroups depends on their some characteristics, for example, complexities. Proceeding from possible substantive interpretations, the following versions of the cost functional are suggested in [2]:

$$P(g_1, \ldots, g_k) = \left[C(g_1) + \ldots + C(g_k) - \max(C(g_1), \ldots, C(g_k))\right]^\beta; \tag{1}$$

$$P(g_1, \ldots, g_k) = \left[C(g_1) + \ldots + C(g_k)\right]^\beta; \tag{2}$$

$$P(g_1, \ldots, g_k) = \frac{C(g)}{\max(C(g_1), \ldots, C(g_k))} - 1; \tag{3}$$

$$P(g_1, \ldots, g_k) = \sum_{i=\overline{1,k}} (C(g) - C(g_i)), \tag{4}$$

where the group $g = g_1 \cup \ldots \cup g_k$ is arranged from subgroups $g_1, \ldots, g_k$, $\beta \in (0; +\infty)$.

It is obvious that the functionals (1) and (2) are monotonic, while the functionals (3) and (4) are not monotonic. To prove the subsequent assertions, we use inequalities that are readily proved by induction on n:

$$(x_1 + \ldots + x_n)^z \geq x_1^z + \ldots + x_n^z \quad \text{for any} \quad x_1 \geq 0, \ldots, x_n \geq 0 \quad \text{for} \quad z \geq 1, \tag{5}$$

$$(x_1 + \ldots + x_n)^z \leq x_1^z + \ldots + x_n^z \quad \text{for any} \quad x_1 \geq 0, \ldots, x_n \geq 0 \quad \text{for} \quad z \leq 1. \tag{6}$$

Assertion 2. *The functional* (1) *for $\beta \leq 1$ is concave; for $\beta \geq 1$, it is convex; and for $\beta \geq 1$ and $\alpha\beta \geq 1$, it is substantially convex.*

Proof. We will consider an admissible array $\{f_1, \dots, f_k\}$ and an arbitrary subarray $\{g_1, \dots, g_r\} \subset \{f_1, \dots, f_k\}$, $2 \le r < k$. Let $\{h_1, \dots, h_{k-r}\} = \{f_1, \dots, f_k\} \backslash \{g_1, \dots, g_r\}$; $g = g_1 \cup \dots \cup g_r$; $x_i = C(g_i)$, $i = \overline{1, r}$; $y_j = C(h_j)$, $j = \overline{1, k-r}$; $x = \max(x_i)$; and $y = \max(y_j)$. We will denote by P_1 the left side and by P_2 the right side in the inequalities of Definition 12: $P_1 = (X + Y - \max(x, y))^\beta$, $P_2 = (X - x)^\beta + (C(g) + Y - \max(y, C(g)))^\beta$, where$X = x_1 + \dots + x_r$ and $Y = y_1 + \dots + y_{k-r}$. With $\beta \le 1$, in view of (6), $P_2 \ge (X + Y + C(g) - x - \max(y, C(g)))^\beta$. To prove the inequality $P_1 \le P_2$, it remains to show that $x + \max(y, C(g)) \le C(g) + \max(x, y)$. For $y \le C(g)$, the inequality is obviously fulfilled. For $y > C(g)$, the inequality will be rewritten in the form $x + y \le C(g) + \max(x, y)$, which stems from $x \le C(g)$. Thus, $P_1 \le P_2$ is met for $\beta \le 1$, i.e., the functional (1) is concave.

Let $\beta \ge 1$. We denote $x_i = C(f_i)$, $i = \overline{1, k}$. Without the loss of generality, we assume that $x_1 = \max(x_1, \dots, x_k)$. Let us set $\{g_1, g_2\} = \{f_1, f_2\}$ and $\{h_1, \dots, h_{k-2}\} = \{f_3, \dots, f_k\}$. We have $P_1 = (x_2 + \dots + x_k)^\beta$ and $P_2 = x_2^\beta + (x_3 + \dots + x_k)^\beta$. In view of (5), $P_1 \ge P_2$ is met for $\beta \ge 1$, i.e., the inequality (a) of Definition 12 is valid; consequently, the functional (1) is convex.

Let $\beta \ge 1$ and $\alpha\beta \ge 1$. We will consider an admissible array $\{g_1, g_2\}$ of nonelementary groups.

Let $C(g_1) \le C(g_2)$. We denote by P_1 the left side and by P_2 the right side of the inequality (a) of Definition 14: $P_1 = P(g_1, g_2)$; for $a \in g_1$, $P_2 = P(g_1 \setminus \{a\}, g_2) + P((g_1 \setminus \{a\}) \cup g_2, \{a\})$. If $g_1 \setminus \{a\} \subset g_2$, then $P_2 = P(g_2, \{a\})$. If $g_1 \setminus \{a\} \not\subset g_2$ but $\{a\} \subset g_2$, then $P_2 = P(g_1 \setminus \{a\}, g_2)$. In both cases, $P_1 \ge P_2$ on account of the monotonicity of (1). If $g_1 \setminus \{a\} \not\subset g_2$, $\{a\} \not\subset g_2$, then we denote $x = C(g_1)$, $y = C(g_1 \setminus \{a\})$, and $z = C(\{a\})$. We have $P_1 = x^\beta$ and $P_2 = y^\beta + z^\beta$. In view of $x = (y^{1/\alpha} + z^{1/\alpha})^\alpha$, the inequality $P_1 \ge P_2$ has the form $(y^{1/\alpha} + z^{1/\alpha})^{\alpha\beta} \ge (y^{1/\alpha})^{\alpha\beta} + (z^{1/\alpha})^{\alpha\beta}$. The last inequality is met in view of (5) with $\alpha\beta \ge 1$. In the case of $C(g_1) \ge C(g_2)$, the fulfillment of the inequality (b) of Definition 14 is proved in a similar way, accurate to the replacement of g_1 by g_2. Consequently, with $\beta \ge 1$ and $\alpha\beta \ge 1$, the functional (1) is substantially convex.

Assertion 3. *The functional* (2) *is concave when* $\beta \le 1$ *and when* $\beta > 1$ *and* $\alpha \ge 1$, *it is concave on arrays of nonintersecting groups.*

Proof. We will examine an admissible array $\{f_1, \dots, f_k\}$ and an arbitrary subarray $\{g_1, \dots, g_r\} \subset \{f_1, \dots, f_k\}$, $2 \le r < k$. Let $\{h_1, \dots, h_{k-r}\} = \{f_1, \dots, f_k\} \setminus \{g_1, \dots, g_r\}$, $g = g_1 \cup \dots \cup g_r$. We put $X = C(g_1) + \dots + C(g_r)$ and $Y = C(y_1) + \dots + C(y_{k-r})$. We denote by P_1 the left side and by P_2 the right side in the inequalities of Definition 12: $P_1 = (X + Y)^\beta$ and $P_2 = X^\beta + (C(g) + Y)^\beta$. When $\beta \le 1$, in view of (6) we have $P_1 \le X^\beta + Y^\beta \le P_2$. If among $g_1, \dots, g_r$ there are no intersecting groups, then $C(g) = (C(g_1)^{1/\alpha} + \dots + C(g_r)^{1/\alpha})^\alpha$. For $\alpha \ge 1$, in view of (5) we have $C(g) \ge X$ and, hence, $P_2 \ge (X + Y)^\beta = P_1$, which thus proves the assertion.

Assertion 4. *The functional* (3) *is substantially convex.*

Proof. First, we will prove the convexity. Let us consider an admissible array $\{f_1, \dots, f_k\}$ and subarrays $\{g_1, g_2\} = \{f_1, f_2\}$ and $\{h_1, \dots, h_{k-2}\} = \{f_3, \dots, f_k\}$. Without the loss of generality, we assume that $C(f_1) = \max(C(f_1), \dots, C(f_k))$. We denote $x = C(f_1 \cup \dots \cup f_k)$; $y = C(g)$, where $g = g_1 \cup g_2$; $z = C(f_1)$; and P_1 and P_2 are the left and the right side, respectively, of the inequality (a) of Definition 12. Then, $z \le y \le x$, $P_1 = x/z - 1$, and $P_2 = y/z - 1 + x/y - 1$, and so we can write

$$P_1 - P_2 = x/z - 1 - y/z + 1 - x/y + 1 = (xy + yz - y^2 - xz)/yz.$$

We denote $\xi(x) = xy + yz - y^2 - xz$. We will perform the differentiation: $\xi'(x) = y - z \ge 0$ in view of $y \ge z$. Next, $\xi(y) = y^2 + yz - y^2 - yz = 0$ and, hence, for all $x \ge y$ we have $\xi(x) \ge 0$, i.e., $P_1 \ge P_2$. Thus, the functional (3) is convex.

We will consider an admissible array $\{g_1, g_2\}$ of nonelementary groups. Let $C(g_1) \leq C(g_2)$. We denote $x = C(g_1 \cup g_2)$; $z = C(g_2)$; and P_1 and P_2 are the left and the right side of the inequality (a) of Definition 14. We have $P_1 = P(g_1, g_2) = x/z - 1$. Let us consider $a \in g_1$, in which case $P_2 = P(g_1 \setminus \{a\}, g_2) + P((g_1 \setminus \{a\}) \cup g_2, \{a\})$. If $g_1 \setminus \{a\} \subset g_2$, then $P_2 = P(g_2, \{a\}) = P_1$. If $g_1 \setminus \{a\} \not\subset g_2$ but $\{a\} \subset g_2$, then $P_2 = P(g_1 \setminus \{a\}, g_2) = P_1$. If $g_1 \setminus \{a\} \not\subset g_2$ and $\{a\} \not\subset g_2$, then we denote $y = C((g_1 \setminus \{a\}) \cup g_2)$. We have $P_2 = y/z - 1 + x/y - 1$. It is shown above that $P_1 \geq P_2$. In the case of $C(g_1) \geq C(g_2)$, the fulfillment of the inequality (b) of Definition 14 is proved in a similar way to an accuracy of the replacement of g_1 by g_2. Consequently, the functional (3) is substantially convex.

Assertion 5. *The functional* (4) *is convex.*

Proof. We will consider an admissible array $\{f_1, \ldots, f_k\}$ and subarrays $\{g_1, g_2\} = \{f_1, f_2\}$ and $\{h_1, \ldots, h_{k-2}\} = \{f_3, \ldots, f_k\}$. We introduce the designations: $g = g_1 \cup g_2$; $y = C(g)$; $x_i = C(f_i)$, $i = \overline{1,k}$; $x = C(f_1 \cup \ldots \cup f_k)$; and P_1 and P_2 are the left and the right side of the inequality (a) of Definition 12. Then, $P_1 = kx - \sum\limits_{i=\overline{1,k}} x_i$, $P_2 = 2y - x_1 - x_2 + (k-1)x - y - \sum\limits_{i=\overline{3,k}} x_i$, and $P_1 - P_2 = x - y \geq 0$. Thus, the functional (4) is convex.

5. CONTROL OF THE ORGANIZATIONAL SYSTEM

In response to changes in the external medium, the system may undergo structural changes that require expenditures for the reorganization. In [3], the cost $\rho(G', G'')$ of the reorganization of the graph G' to the graph G'' is defined, i.e., the cost of the rearrangement of the organization G' to the organization G'' (in particular, the cost $\rho(\varnothing, G)$ of the establishment of the organization G "from zero"). The reorganization cost is estimated on the basis of the known quantities: $\rho'(a, g)$, which is the cost of the exclusion of the performer a from an arbitrary group g, $a \in g$, and $\rho''(a, g)$, which is the cost of the inclusion of a performer in an arbitrary group g, $a \notin g$. Then, G' rearranges to G'' by way of the sequential exclusion and inclusion of performers. The reorganization cost $\rho(G', G'')$ is equal to a minimum total cost of all exclusions and inclusions (in greater detail, see [3]). If we put $\rho''(a, g) = C(a)$, then we obtain the complexity $C(G) = \rho(\varnothing, G)$ of the organization G. In the text presented below, we consider the notion of a control that determines the total profit on the interval $\overline{1,T}$.

We assume that the cost functional P is invariable. The aggregate of invariable parameters of the external medium, which are given in the Introduction, namely, $I_1, \ldots, I_q$, $e_1, \ldots, e_r$, W, and the functional P will be denoted by E. The changing parameters of the external medium, which are known before the beginning of the unit time t, $t = \overline{1,T}$ are the following: the behavior of the labor market, $\widetilde{\mathbf{A}}^t = \left\{\widetilde{A}^t, \ldots, \widetilde{A}^{t-1}\right\}$, in the preceding periods of time and variations of the prices and the volume of the demand, $\mathbf{p}^t = \{\mathbf{p}^1, \ldots, \mathbf{p}^t\}$, $\mathbf{v}^t = \{\mathbf{v}^1, \ldots, \mathbf{v}^t\}$.

Before the beginning of the time t, $t = \overline{1,T}$, the system has the set A^{t-1} of performers organized with the aid of the graph G^{t-1}. If the interval T under examination is rigidly preset, then we put $G^0 = (\varnothing, \varnothing)$, which is an empty graph (it is necessary to create the initial organization beginning with zero). But if we investigate a potentially infinite interval (T is much larger than the period of oscillations of the external medium), then we put $G^0 = G^1$ (the initial organization is already created).

Definition 16. The correct array of controllable parameters for the unit time t, $t = \overline{1,T}$ will be called the following: the array of performers $A^t \subseteq A^{t-1} \cup \widetilde{A}^{t-1}$; the plans $\mathbf{y}^t$ and $\mathbf{x}^t(a)$ for all $a \in A^t$; the correct array $\mathbf{f}^t$ of groups; and the graph G^t of the organization of groups of the array $\mathbf{f}^t$.

By controlling the system, it is necessary to define the array of controllable parameters before the beginning of a unit time, proceeding from the information E, $\widetilde{\mathbf{A}}^t$, $\mathbf{p}^t$, and $\mathbf{v}^t$ on the external medium, the available structure G^{t-1} and performers A^{t-1}, the time t, and the length of the interval T under examination.

Definition 17. The control of the organizational system will be called an arbitrary mapping of $\Psi(E, \widetilde{\mathbf{A}}^t, \mathbf{p}^t, \mathbf{v}^t, A^{t-1}, G^{t-1}, t, T)$ of its arguments into the correct array of controllable parameters.

Definition 18. The result of a control of the system will be called the quantity

$$R\left(E, \widetilde{\mathbf{A}}^T, \mathbf{p}^T, \mathbf{v}^T, \Psi\right) = \frac{1}{T} \sum_{t=\overline{1,T}} \left[(\mathbf{p}^t, \mathbf{y}^t) - P(G^t) - \rho(G^{t-1}, G^t)\right],$$

where $(\mathbf{p}^t, \mathbf{v}^t)$ is the total gain of the system in the unit time t, $\rho(G^{t-1}, G^t)$ are expenditures for the reorganization of the structure G^{t-1} to G^t, and $P(G^t)$ are expenditures for operation of the system with the structure G^t for a unit time.

The result of a control of the system—the mean profit in the time interval $\overline{1,T}$—depends on the control Ψ itself and on a change in the parameters E, $\widetilde{\mathbf{A}}^T$, $\mathbf{p}^T$, and $\mathbf{v}^T$ of the external medium.

Definition 19. An optimal control will be called a control

$$\Psi = \arg\max R\left(E, \widetilde{\mathbf{A}}^T, \mathbf{p}^T, \mathbf{v}^T, \Psi'\right),$$

where the maximum is taken over all controls Ψ'.

If some of the trajectories of the parameters $\widetilde{\mathbf{A}}^T, \mathbf{p}^T$, and $\mathbf{v}^T$ of the external medium are unknown, but their probability distribution is known, then we will redefine the optimal control.

Definition 20. *An optimal control in the mean will be called a control*

$$\Psi = \arg\max \overline{R}\left(E, \widetilde{\mathbf{A}}^T, \mathbf{p}^T, \mathbf{v}^T, \Psi'\right),$$

where the mean value is taken over those parameters of the external medium for which only the probability distribution is known and the maximum is taken over all controls Ψ'.

The problem on an optimal control is rather complex for an analytical solution. However, if the control is estimated effectively, then the result of a control of the system with the prescribed trajectory of parameters of the external medium can also be estimated effectively. The result of a control in the mean can be replaced by the sample mean. Thus, if we consider an array of effectively computable controls, then an optimal one can be found among them. If additional constraints on the choice of a control are specified, then the controls that do not satisfy them can be excluded, which will lead to a conditionally optimal control. For example, under constraints on a resource, some controls that involve inadmissible losses at individual instants of time must be left out of consideration even in the case of their optimality. We will define some kinds of controls.

Definition 21. A control will be called a control with the constant composition if $A_1 = A_2 = \ldots = A_T = A_0$ is met.

Definition 22. A control will be called a control with the trivial planning if the correct plans $\mathbf{y}^t$ of the output and the correct plans $\mathbf{x}^t(a)$ of jobs are defined proceeding from the maximization of the "gross profit" $(\mathbf{p}^t, \mathbf{y}^t) - \sum_{a \in A^t} p(a, \mathbf{x}^t(a))$ for the unit time t.

If the expenditures $p(a, \mathbf{x}^t(a))$ depend linearly on components of the vector $\mathbf{x}^t(a) = (x_1^t(a), \dots, x_r^t(a))$ of the plan of jobs, then, taking into account the linear constraints on $\mathbf{y}^t$ and $\mathbf{x}^t(a)$, the determination of the trivial plan is the problem of linear programming.

The correct array $\mathbf{f}^t$ of groups can be defined in the following way. We distribute the volume $s_1(a)x_1^t(a)$ of the job e_1 carried out by the performer a for the output of the first product, in the case of the surplus, we distribute the volume of the job for the output of the second product, etc. Then, we deal in a similar way with the volume of the job e_2 done by the performer a, etc. On distributing all jobs of the performer a over products, we pass on to the next performer.

Definition 23. A control will be called a control with the trivial grouping if the correct array of groups is specified by the foregoing method.

In the case of the complex investigation of the model in the foregoing statement, it is possible to analyze the effect of all controllable parameters on the result of the control. If the aim of the investigation is to control structural changes, then we can consider only the problem of the choice of a structure. For example, we can give the following definition.

Definition 24. The control with the constant composition and the trivial planning and grouping will be called the control of a structure.

The controls of a structure differ only in the choice of organization graphs G^t, $t = \overline{1,T}$. We can give the following examples of the control of a structure.

The control of a minimum cost minimizes the mean operating cost $\left(\sum_{t=\overline{1,T}} P(G^t)\right)/T$, i.e., finds an optimal organization at each $t = \overline{1,T}$. If $\rho(G', G'') \equiv 0$, then the control of the structure of a minimum cost is optimal.

The control of a minimum complexity minimizes the mean complexity $\left(\sum_{t=\overline{1,T}} C(G^t)\right)/T$ of an organization. The simultaneous organization has a minimum complexity among all organizations of the prescribed array of groups (see [3]). Consequently, the control of a minimum complexity exists if all organizations $G^1, \dots, G^T$ are simultaneous.

6. CONCLUSIONS

The model is built up that enables us to compare the effectiveness of the various controls of structural changes in the organizational system. In the case of an optimal control of the structure, we reach the best balance between the mean expenditures $\left(\sum_{t=\overline{1,T}} P(G^t)\right)/T$ for operation of the system and the mean expenditures $\left(\sum_{t=\overline{1,T}} \rho(G^{t-1}, G^t)\right)/T$ for the reorganization (the latter expenditures are defined by the ability of the structure to react in a flexible manner to changes in the external medium). If the cost of the reorganization is zero, then it remains to minimize expenditures for the functioning, which is achieved by means of a minimum cost control, for the estimation of which it is necessary to solve the problem for an optimal organization.

For substantially convex functionals there exists an optimal sequential organization (see Theorem 4), which can be found with aid of the algorithms constructed in [2], i.e., the proved theorem in combination with the algorithms [2] solves in the exhaustive manner the problem of an optimal organization for substantially convex functionals.

For monotonic functionals there exists an optimal tree of the organization of one group (see Theorem 1), which can be found by means of the algorithms constructed in [4]. If, apart from the monotonicity, the functional displays the convexity on the arrays of nonintersecting groups, then there exists an optimal 2-tree of the organization of one group (see the Corollary to Theorem 2), so that the problem for an optimal tree is simplified (see the algorithms in [4]). If, apart from the monotonicity, the functional displays the concavity on the arrays of nonintersecting groups, then the simultaneous organization of one group is optimal (see Theorem 3). These results in combination with the algorithms in [4] solve the problem of an optimal organization of one group for various classes of functionals.

In this work, we cite examples of the cost functionals (1)–(4). We proved a substantial convexity of the functional (1) for $\beta \geq 1$ and $\alpha\beta \geq 1$ (see Assertion 2) and of the functional (3) at any α (see Assertion 4), which enables us to solve the general problem of an optimal organization. The next results solve the question as to an optimal organization of one group for the functionals (1)–(3).

For the functional (1), the simultaneous organization of one group is optimal when $\beta \leq 1$ (in view of its concavity, see Assertion 2); when $\beta \geq 1$, the sequential organization of one group is optimal (see [2]), which is found in [4]. When $\beta \leq 1$ or when $\beta > 1$ and $\alpha \geq 1$, the simultaneous organization of one group is optimal for the functional (2) (in view of its concavity on the arrays of nonintersecting groups, see Assertion 3); in the remaining domain, it is possible to find the optimal organization of one group with the aid of the algorithms of the search for an optimal tree (in view of the monotonicity of (2)). For the functional (3), the sequential organization of one group is optimal (in view of its substantial convexity, see Assertion 4), which is found in [4].

Thus, the analysis of the belonging of a functional to various classes permits us to find, relying on Theorems 1–4, the form of an optimal organization, which is illustrated by the examples of the functionals (1)–(4) (see Assertions 2–5).

The modeling of an organizational system offers the possibility of analyzing the effect of various parameters of the external medium on the effectiveness of a control of the structure. For example, the calculation of the mean complexity $\left(\sum_{t=\overline{1,T}} C(G^t)\right)/T$ of an organization with various controls from the prescribed array allows us to verify the regularity observed in practice: in the case of rigid (intensive) external changes, a simple structure of the system (the simultaneous organization) is optimal, which becomes more complicated as the external actions ease off. By the intensity of external changes can be meant, for example, the Euclidean distance between price vectors $\mathbf{p}^{t-1}$ and $\mathbf{p}^t$ or between demand volumes $\mathbf{v}^{t-1}$ and $\mathbf{v}^t$.

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This paper was recommended for publication by D.A. Novikov, a member of the Editorial Board