DYNAMIC ACTIVE SYSTEMS WITH FAR-SIGHTED ELEMENTS.
I. A DYNAMIC MODEL OF AN ACTIVE SYSTEM

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We consider a dynamic model of an active system in which the choice of states by the active element in one period affects the state set in the next period. We investigate the functioning modes of the dynamic system and analyze the ways to incorporate the future in the performance evaluation criteria of the active element and the headquarters.

1. INTRODUCTION

In order to improve the efficiency of control of the socialist economy, we must take into consideration the human element in the control system. The presence of the human element (groups of people) in the system endows it with certain "activity," reflected in goal-directed actions in the functioning stages. In particular, this includes the ability to consider the aftereffects of various decisions and to appropriately modify the actions. The functioning mechanisms of systems which allow for the human factor are the subject of the theory of active systems [1]. Studies in AS theory have developed and investigated a number of models of social-economic objects and examined problems of analysis and design of functioning mechanisms. Interesting results relating to the functioning mechanisms ensure truthful reporting in the system and agreement of actual performance with planned targets. Most of these results, however, have been obtained for static models of active systems.

A common feature of control in economic systems is the repetitive nature of decision-making situations at each stage of the process. An example is provided by problems in which decisions must be made repeatedly. Such problems include the construction of a detailed operations plan of an enterprise at each stage of its operation on the basis of externally set directives, the problem of optimal allocation of resources, inventory management, etc. If we allow for the fact that the decision maker is endowed with certain far-sightedness, i.e., considers the consequences of decisions on future possible states, the functioning of such a system must be analyzed with the aid of dynamic models of repeating situations.

The corresponding problems can be solved, in particular, by the methods of repetitive games [2, 3]. The mathematical models of repetitive game situations were studied in [4-6] with emphasis on the existence conditions of stable strategies under various behavioral assumptions and in n-person games with varying value of the unit of payment. In this study, we examine the functioning of a dynamic AS with far-sighted elements.

2. THE MODEL

Our model represents a two-level hierarchical system which consists of the headquarters and n subordinated subsystems [active elements (AE) or divisions]. For a detailed description of the interdependences between the elements and the actions of the headquarters and the divisions in a static AS model, see [7]. Here, as in [1], the state of a division is called actual performance, and the "desired" values of the components of actual performance constitute the plan or the targets. The active elements choose their states in each operating period.

Suppose that the headquarters sets the divisional plans for each period of operation. The plan set by the headquarters for the i-th AE in the j-th operating period will be denoted by π_{ij}, and the actual performance of the i-th AE in the j-th period by y_{ij}. If the headquarters sets the divisional plans for T operating periods at a time, these T periods jointly constitute the planning horizon, and the sequence of the plans is the planning path.

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In modern planning practice, we have five-year plans with target breakdown for each year or annual plans with quarterly or monthly breakdown. In the former case, the five-year period is the planning horizon and each year is the operating period, whereas in the latter case the planning horizon is one year and the operating period is a quarter or a month. For the static AS models investigated in [1-7], T = 1 and the planning horizon coincides with the operating period.

We denote the planning path of the i-th AE by $\pi_i^T(1) = \{\pi_{ij}\}$, j = 1, ..., T, and the state path (or the actual performance path) over the planning horizon by $y_i^T = \{y_{ij}\}$, j = 1, ..., T. By $y_i^p(k) = \{y_{ij}\}$, j = k, k + 1, ..., p and $\pi_i^p(k) = \{\pi_{ij}\}$, j = k, k + 1, ..., p we denote the sections of the performance paths and the planning path between operating periods k and p.

The divisions choose the actual performance in each operating period from the feasible performance set. Because of the "dynamic property" of our AS model, the choice of actual performance by the i-th AE in the j-th operating period affects the feasible performance set in period j + 1.

We denote the feasible performance set of the i-th AE in the j-th operating period by $B_{ij}(\pi_{ij}, y_{ij-1})$. The actual performance y_{ij} satisfies $y_{ij} \in B_{ij}(\pi_{ij}, y_{ij-1})$. The performance path selected by the i-th AE over the planning horizon clearly should satisfy the condition

$$y_i^T(1) \in \prod_{j=1}^{T} B_{ij}(\pi_{ij}, y_{ij-1}).$$

The choice of actual performance by the AE in each operating period is aimed at satisfying the divisional preferences. We assume that the preferences of the i-th AE in the j-th operating period are quantitatively expressed by the function $f_{ij}(\delta_{ij}, y_{ij})$, and the divisional behavior is aimed at maximizing the value of this function [7]. This function depends on the planned targets and on the selected performance since, in practice, the activity evaluation of industrial objects, and hence also the incentives they receive in each operating period, are determined by comparing the planned measures with the activity measures.

We also assume that the preferences of the system as a whole are identical to the headquarters preferences. Quantitatively, the headquarters preferences in operating period j are expressed by the function $\Phi_j(\pi_{1j}, \ldots, \pi_{nj}, y_{1j}, \ldots, y_{nj})$, where n is the number of active elements in the system. The choice of actual performance by all the AE quantitatively determines the payoff to the headquarters.

3. THE FUNCTIONING OF THE DYNAMIC MODEL

In active system theory, each operating period includes three stages: the data generation stage, the planning stage, and the performance stage. For the static model, detailed description of each stage and the actions of the divisions and the headquarters will be found in [7].

In dynamic AS models, the planning horizon consists of several operating periods. If the headquarters or the divisions are far-sighted, i.e., capable of foreseeing the impact of their performance decisions on future operating periods, the range of possible system functioning modes is broadened.

If the planning horizon includes a data-generation stage, a planning stage, and a performance stage, which are common for all the operating periods, there will be no fundamental difference between the dynamic and the static AS models. Formally identifying the planning horizon with the common operating period, we can study the system behavior on these common periods, without attempting to analyze the processes within each period. Thus, with this functional organization, the behavior of the model actually reduces to the standard static AS model [1, 7].

Now assume that each operating period includes all the three stages, and the headquarters sets the divisional plans only for the current operating period, ignoring the possible AS states in future periods. This activity corresponds to headquarters without far-sightedness (near-sighted or myopic behavior). The divisions in this case are incapable of fully assessing the impact of their performance decisions in the current period on the feasible performance set in the subsequent periods. This is due to the fact the feasible performance set of the i-th AE in the j-th operating period is determined by the plan, and in the assumed framework the plan π_{ij} is not generated in period $(j - 1)$. If we assume that the divisions make no predictions about future targets in subsequent operating periods, we conclude that the functioning of the system over the planning horizon can be studied by examining the sequential behavior of the AS over all the operating periods.

If the headquarters is far-sighted, the initial planning path set at the beginning of the planning horizon may be revised or updated in future operating periods based on information about the actual AE performance. The revised planning path as observed at the end of the planning horizon may be entirely different from the initially chosen path. For far-sighted AE, constant revisions of the planning path by the headquarters naturally make the prediction of future states more difficult.

Now assume that the planning path is set for T operating periods: the data-generation stage and the planning stage are common for the entire planning horizon, but the performance stage is repeated in each operating period. Also assume that the planning path remains unchanged over the entire operating period. Thus, the planning horizon consists of one data generation stage, one planning stage, and T performance stages. In what follows, we assume this particular functioning mode of the dynamic AS.

4. INCORPORATING THE FUTURE IN THE PERFORMANCE EVALUATION CRITERIA OF THE ACTIVE ELEMENTS AND THE HEADQUARTERS

The value of the function $f_{ij}(\pi_{ij}, y_{ij})$ provides a quantitative expression of the preferences of the i-th AE only in the j-th operating period. Therefore, the choice of $f_{ij}(\pi_{ij}, y_{ij})$ as the AE objective function is justified if the divisional decisions in the current operating period do not affect the future periods, or if the AE is incapable of allowing for the consequences of its performance decisions. The i-th AE attains the maximum satisfaction in the j-th operating period if it chooses the actual performance by solving the problem

$$f_{ij}(\pi_{ij}, y_{ij}) \to \max_{y \in B_{ij}(\pi_{ij}, y_{ij-1})}, \quad j=1, \ldots, T. \tag{1}$$

In what follows, we assume that the function $f_{ij}(\pi_{ij}, y_{ij})$ attain their maxima on the sets B_{ij}, $i = 1, \ldots, n$, $j = 1, \ldots, T$.

The objective function of a far-sighted AE in the j-th operating period is representable in the form

$$\varphi_{ij}=f_{ij}(\pi_{ij}, y_{ij})+\sum_{k=j+1}^{j+N} f_{ik}(\pi_{ik}, y_{ik}). \tag{2}$$

The number of future periods N which are incorporated by the AE in the objective function determines its degree of far-sightedness [7]. For a myopic AE, $N = 0$.

In our dynamic AS model, both far-sighted and myopic AE select their actual performance in each operating period. If $\hat{y}_{ij}$ is the solution of problem (1), then its performance path $\hat{y}_i^T(1)$ is made up of a sequence of solutions of problem (1) for $j = 1, \ldots, T$.

A far-sighted AE chooses the actual performance for the current period and also predicts the performances in the future periods, attempting to maximize the objective function (2). Thus, in the j-th operating period the i-th AE solves the problem

$$\sum_{k=j}^{j+N} f_{ik}(\pi_{ik}, y_{ik}) \to \max_{y_i^{j+N}(j) \in \prod_{k=j}^{j+N} B_{ik}(\pi_{ik}, y_{ik-1})}.$$

The solution of this problem is a collection of vectors $\hat{y}_{ij}, \tilde{y}_{ij+1}, \ldots, \tilde{y}_{ij+N}$. The vector $\hat{y}_{ij}$ is the actual performance of the AE in the j-th operating period, and the vectors $\tilde{y}_{ij+1}, \ldots, \tilde{y}_{ij+N}$ are the predicted performance for the future periods from $j + 1$ to $j + n$.

The problem solved by the AE in operating period $j + 1$ has the form

$$\sum_{k=j+1}^{j+1+N} f_{ik}(\pi_{ik}, y_{ik}) \to \max_{\substack{y_i^{j+1+N}(j+1) \in \prod_{k=j+1}^{j+1+N} B_{ik}(\pi_{ik}, y_{ik-1}) \\ y_{ij}=\hat{y}_{ij}}} .$$

The resulting collection of vectors $y_{ij+1}, \overset{*}{y}_{ij+2}, \ldots, \overset{*}{y}_{ij+1+N}$ gives the actual performance y_{ij+1} in period j + 1 and the predicted performances $\overset{*}{y}_{ij+2}, \ldots, \overset{*}{y}_{ij+1+N}$ for the future periods. Here, the functioning of the AS is considered separately for each planning horizon, and it is assumed that the choice of the actual performance by the AE for any operating period in one planning horizon does not affect the feasible performance set in another planning horizon. If the degree of far-sightedness of the AE is such that j + N > T, the problem (2) solved by the AE in period j may be rewritten as

$$\sum_{k=j}^{T} f_{ik}(\pi_{ik}, y_{ik}) \to \max_{y^T(j) \in \prod_{k=j}^{T} B_{ik}(\pi_{ik}, y_{ik-1})} .$$

Thus, over a planning horizon consisting of T operating periods, the objective of a far-sighted AE may be written as

$$\varphi_{ij} = \sum_{k=j}^{p_j} f_{ik}(\pi_{ik}, y_{ik}),$$

where

$$p_j = \begin{cases} j+N, & \text{if} \quad j+N \leqslant T, \\ T, & \text{if} \quad j+N > T. \end{cases}$$

The problem solved by the i-th AE in the j-th operating period takes the form

$$\sum_{k=j}^{p_j} f_{ik}(\pi_{ik}, y_{ik}) \to \max_{y_i^{p_j}(j) \in \prod_{k=j}^{p_j} B_{ik}(\pi_{ik}, y_{ik-1})} . \tag{3}$$

If the AE assign different values to the function f in different operating periods when choosing the actual performance for the j-th period and making predictions for the future, then the problem (2) should be rewritten as

$$\sum_{k=j}^{p_j} \delta_{ik} f_{ik}(\pi_{ik}, y_{ik}) \to \max_{y_i^{p_j}(j) \in \prod_{k=j}^{p_j} B_{ik}(\pi_{ik}, y_{ik-1})} . \tag{4}$$

As a rule, we set $\delta_{ij} = 1$ when solving problem (4). In this case, the components of the vector $\{\delta_{ik}\}$ determine the weight of the function f of the i-th AE in future operating periods in relation to its value in the current period. In the economic literature, the dependence of δ_{ik} on the index of the operating period is mainly represented in the form $\delta_{ik} = \delta_1^{k-1}$ or $\delta_{ik} = e^{-\alpha_i k}$ (α_i is a constant coefficient). In this case, we say that the dependence of δ_{ik} on k reflects the degree of incorporating the future in the current decisions. The coefficient δ_{ik} is known as the discounting factor [2].

In what follows, we call the vector $\{\delta_{ik}\}$ the far-sightedness distribution of the i-th AE. The components of this vector may be "rigidly tied" to operating periods or may change when solving problem (4) in different operating periods.

Let us find the form of the problems that the AE solves in each operating period when the far-sightedness distribution varies. Suppose that in the first operating period the far-sightedness distribution is $\delta_{i1}^{(1)}, \delta_{i2}^{(1)}, \ldots, \delta_{ij}^{(1)}, \ldots, \delta_{ip_1}^{(1)}$, in the second period it is $\delta_{i2}^{(2)}, \ldots, \delta_{ij}^{(2)}, \ldots, \delta_{ip_2}^{(2)}$, and in the j-th period $\delta_{ij}^{(j)}, \ldots, \delta_{ip_j}^{(j)}$. Then, the objective function of the i-th AE in the j-th operating period is

$$\sum_{k=j}^{p_j} \delta_{ik}^{(j)} f_{ik}(\pi_{ik}, y_{ik}).$$

The AE problem is correspondingly written in the form

$$\sum_{k=j}^{p_j} \delta_{ik}^{(j)} f_{ik}(\pi_{ik}, y_{ik}) \to \max_{y_i^{p_j(j)} \in \prod_{k=j}^{p_j} B_{ik}(\pi_{ik}, y_{ik-1})} . \tag{5}$$

If the degree of incorporating the future in decision making is constant over the operating periods, the components of the far-sightedness distribution vector in operating period j are expressible in the form

$$\delta_{ik}^{(j)} = \delta_{ik-j+1}^{(1)}, \quad k=j, \ldots, p_j.$$

Let $p_j = T$ and let the components of the far-sightedness distribution vector be $\delta_{ik} = \delta_i^{k-1}$ or $\delta_{ik} = e^{-\alpha_i k}$. It is easy to show that the actual performance in the j-th operating period and the predicted states for the future periods obtained by solving problem (4) coincide with the actual performance and the predictions obtained by solving the problem (5). Indeed, the objective function (4) differs from the objective function (5) only in the coefficient δ_i^{k-1}, which does not affect the solution. The objective function (4) characterizes the case when the components δ_i^{k-1} are "rigidly linked" to the operating periods, whereas the objective function (5) represents the case of variable far-sightedness distribution over the operating periods, while the degree of incorporating the future remains the same. Therefore, if the components of the far-sightedness distribution vector have the form $\delta_{ik} = \delta_i^{k-1}$ or $\delta_{ik} = e^{-\alpha_i k}$, the actual performance and the predictions obtained for the components δ_{ik} "rigidly linked" to operating periods are identical to the actual performance and predictions obtained with variable far-sightedness distribution and constant degree of incorporating the future.

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