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CONTROL IN SOCIAL ECONOMIC SYSTEMS

Algorithms to Seek the Optimal Structure of the Organizational System

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Abstract—A socio-economic system of several agents organized in a way to perform the system functions was considered. Each function is realized by a group of agents compiled of some subgroups. A notion of organization of the given groups and its cost was introduced. In the case of invariable environment, design of the optimal system comes to determining a minimal-cost organization. Classes of organizations involving the optimal one were established. Complexity of the problem of determining the optimal organization was estimated, and algorithms to solve it were constructed. Possible strategies for reorganizing the system in varying environment were established, and a model for selecting the optimal strategy was described.

1. INTRODUCTION

The theory of active systems pays much attention to the game-theoretic models of the two-level organization systems consisting of the principal and agents. One of the approaches to the multilevel system lies in decomposing it into a number of two-level systems. Sometimes it allows one to study the constant-structure system [1]. However, the system responds to environmental changes by structural reorganization which cannot be described by the aforementioned models [2]. The problems of choosing the structure either are transferred to the level of problem formulation [3], that is, are not analyzed mathematically, or the problems of job execution by the agents (assignment problem) and determination of the system organization structure are completely separated and the optimal tree is constructed from a rigidly defined class [4]. It seems that the environmental impact on the system structure cannot be modeled within the framework of this formulation.

We assume that the system has at its disposal a collection of agents $a_1, \dots, a_n$ (for example, workers or machines) which can execute some of the elementary jobs $e_1, \dots, e_r$. The aim of the system is to produce some articles from the collection $I_1, \dots, I_q$ corresponding to the industry. Elementary jobs must be executed in order to produce the articles. The collection $e_1, \dots, e_r$ does not include auxiliary operations concerned with system organization, and it is the same for any system of the given industry.

We assume that given are the matrices $T = \{t_{i,j}\}$ and $S = \{s_{i,j}\}$, where $t_{i,j}$ is the number of units e_j required to produce one unit I_i and $s_{i,j}$ is the number of units e_j that can be executed by a_i in a unit time. We also assume that the dimensionless comparable indicators, the complexities $c_1^e, \dots, c_r^e$ of a unit of elementary job (mean labor consumption, consumption of machine time, and so on) are given as well. We suppose that the complexity $C(I_i)$ of producing a unit of product I_i is equal to the sum $C(I_i) = c_1^e t_{i,1} + \dots + c_r^e t_{i,r}$ of complexities of the required elementary jobs. Complexity (potential) $C(a_i)$ of the agent a_i is assumed to be equal to the maximal complexity of the elementary job which a_i can do in unit time: $C(a_i) = \max(c_1^e s_{i,1}, \dots, c_r^e s_{i,r})$.

Let the system produce y_i units of I_i in unit time; at that, $0 \le y_i \le v_i$, where v_i is the maximal volume of product that the system can sell at the market. We denote by $x_{i,j}$ the fraction of time unit which the agent a_i assigns to executing the elementary job e_j. Then, $0 \le x_{i,j} \le 1$, and by

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$z_i = \sum_{j=\overline{1,r}} x_{i,j}$, $0 \le z_i \le 1$, we denote the load of a_i. The relations $\sum_{i=\overline{1,q}} y_i t_{i,j} \le \sum_{k=\overline{1,n}} x_{k,j} s_{k,j}$, $j = \overline{1,r}$, must be satisfied to maintain the balance of the executed and required jobs.

We assume that the cost of supporting the agent a_i over unit time is equal to $p_i^c + z_i p_i^v$, where p_i^c and p_i^v are the constant and variable costs, respectively. We denote by p_i the price of the product I_i. Then, the earnings in unit time will be $V = y_1 p_1 + \ldots + y_q p_q$. The direct costs, that is, the costs of supporting the agents, will be $Z = \sum_{i=\overline{1,n}} (p_i^c + z_i p_i^v)$. The magnitudes of y_i and $x_{i,j}$ can be established, for example, by maximizing the gross margin $V - Z$. Taking into account the linear constraints on y_i and $x_{i,j}$, the last problem, is that of linear programming.

We denote the set of agents by $A = \{a_1, \ldots, a_n\}$. By the group of agents is meant any nonempty subset $f \subseteq A$; the set of all groups is denoted by $F = 2^A \backslash \{\varnothing\}$. The number of agents in a group will be called the group level. Each agent a_i executes elementary jobs of volumes $x_{i,1}s_{i,1}, \ldots, x_{i,r}s_{i,r}$. By distributing them in a way we establish whether a_i is involved in the production of each product. We obtain that the product I_i is manufactured by a subset of agents $f \subseteq A$; here, $f = \varnothing$ for $y_i = 0$, otherwise, $f \in F$. By disregarding the empty sets, we obtain that joint operation of the agents in groups $f_1, \ldots, f_m$, $m \le q$, must be organized to manufacture the products.

For an arbitrary group $f \in F$, we determine its complexity (potential) $C(f) = \left(\sum_{a_i \in f} C(a_i)^{1/\alpha} \right)^\alpha$, where $\alpha \in (0, +\infty)$. For $\alpha = 1$, group complexity is equal to the sum of the complexities of its component agents; for $\alpha > 1$, it is greater than the sum (complicating parameter); for $\alpha < 1$, smaller (simplifying parameter).

We assume that the cost of organizing joint operation of arbitrary subgroups $g_1, \ldots, g_k \in F$ in the group $g = g_1 \cup \ldots \cup g_k$ over time unit is defined by the organization cost functional $P(C(g_1), \ldots, C(g_k), C(g))$ whose arguments are the complexities of subgroups and complexity of the organized group. If P is independent of the last argument, then it will be omitted. The value of P is independent of subgroup permutation. Descriptively, the cost of organizing joint operation is the cost of coordinating actions (control), accounting, transportation, and other overhead charges.

Intermediate groups can be organized for joint operation of the agents in the groups $f_1, \ldots, f_m$. By the cost of organization will be meant the total costs of organization of all groups. The cost of organization (indirect costs) is defined by the particular system structure.

2. SYSTEM ORGANIZATION GRAPH. KINDS OF ORGANIZATIONS

Definition 1. The oriented graph $G = (V, E)$ will be called the organization graph of groups $f_1, \ldots, f_m$ if it satisfies the following conditions:

(a) The vertices correspond to the groups, that is, $V \subseteq F$, $f_1, \ldots, f_m$, $\{a_1\}, \ldots, \{a_n\} \in V$;

(b) $E \subseteq V \times V$, $g \subset h$, $g \ne h$, is satisfied for any edge $(g, h) \in E$ (loop-free graph);

(c) For an arbitrary vertex $g \in V$, we denote by $Q(g) = \{h : (h, g) \in E\}$ the set of vertices from which edges go to g; then, $g = \bigcup_{h \in Q(g)} h$, $Q(\{a_i\}) = \varnothing$ is satisfied for any $g \ne \{a_i\}$, $i = \overline{1,n}$, that is, any group $g \ne \{a_i\}$ is organized from the subgroups of the set $Q(g)$; and

(d) At least one edge goes out of any $g \in V$, $g \notin \{f_1, \ldots, f_m\}$.

By the organization is meant the corresponding graph of organization. The vertices (groups) $\{a_1\}, \ldots, \{a_n\}$ of the graph G will be called the elementary vertices; the nonelementary vertices of G other than $f_1, \ldots, f_m$, the intermediate vertices.

Definition 2. Let the organization $G = (V, E)$ be given. We consider $g \in V \backslash \{a_1\}, \ldots, \{a_n\}$. Let $Q(g) = \{g_1, \ldots, g_k\}$. We label the group g by the cost $R(g)$ of organizing it from the subgroups $g_1, \ldots, g_k$: $R(g) = P(C(g_1), \ldots, C(g_k), C(g))$. By the cost of organizing G will be meant $P(G) = \sum_{g \in V \setminus \{a_1\}, \ldots, \{a_n\}} R(g)$. Organization of G' will be called optimal if $P(G') = \min P(G)$, where all possible organizations of the groups $f_1, \ldots, f_m$ are minimized. By the problem of optimal organization will be meant that of seeking one of the optimal organizations.

A problem similar to that of optimal organization is not encountered among the existing problems of discrete optimization (see, for example, [5]). Its complexity and the algorithms to solve it are discussed below.

Definition 3. The organization $G = (V, E)$ will be called the sequential organization if for any $g \in V \setminus \{a_1\}, \ldots, \{a_n\}$ $Q(g) = \{g \setminus \{a_i\}, \{a_i\}\}$ for some $1 \le i \le n$.

Definition 4. The organization $G = (V, E)$ will be called the r-organization, $r \ge 2$, if for any $g \in V |Q(g)| \le r$.

Definition 5. The organization $G = (V, E)$ will be called the simultaneous organization if $V = \{\{a_1\}, \ldots, \{a_n\}, f_1, \ldots, f_m\}$; at that, $Q(f_i) \subseteq \{\{a_1\}, \ldots, \{a_n\}\}$, $1 \le i \le m$.

The sequential organization is a special case of the 2-organization. The simultaneous organization is unique.

Definition 6. By the complexity $C(G)$ of the organization $G = (V, E)$ of the groups $f_1, \ldots, f_m$ will be meant $C(G) = \left(\sum_{g \in V} C(g) \right) / (C(a_1) + \ldots + C(a_n) + C(f_1) + \ldots + C(f_m))$.

The complexity of simultaneous organization is minimal and equal to unity.

Definition 7. Let G be the organization of the groups $f_1, \ldots, f_m$ of agents $a_1, \ldots, a_n$. By the G-optimal organization of the groups $f_1, \ldots, f_m$, $f_{m+1}, \ldots, f_{m+k}$ of the agents $a_1, \ldots, a_n$, $a_{n+1}, \ldots, a_{n+s}$ will be meant the minimal-cost organization among those that involve G as a subgraph.

3. REQUIREMENTS ON THE COST FUNCTIONAL. POSSIBLE FORMS OF THE FUNCTIONAL

Definition 8. The cost functional will be called uniform if $P(xC_1, \ldots, xC_k, xC) = \xi(x) P(C_1, \ldots, C_k, C)$ is satisfied for any nonnegative numbers x, $C_1, \ldots, C_k$, C, $C \ge \max(C_i)$, where $\xi(x)$ is a monotone nondecreasing function, $\xi(0) = 0$, $\xi(x) > 0$ for any $x > 0$.

Definition 9. The cost functional will be called correct if $P(C, 0, \ldots, 0, C) = 0$ is satisfied for any $C \ge 0$, that is, the cost of organizing a group with zero-complexity groups is zero.

Definition 10. The cost functional $P(C_1, \ldots, C_k, C)$ will be called monotone if (1) $P(C'_1, \ldots, C'_k, C') \ge P(C_1, \ldots, C_k, C)$ is satisfied for any $C'_1 \ge C_1, \ldots, C'_k \ge C_k$, $C' \ge C$, and (2) $P(C_1, \ldots, C_k, C', C'') \ge P(C_1, \ldots, C_k, C)$ is satisfied for any $C' \in [0; +\infty)$, $C'' \ge C$, that is, the cost does not decrease if greater-complexity subgroups are organized and one more subgroup is added to the organized subgroups.

The condition for uniformity of the cost functional ensures independence of the optimal organizations of the scale of complexities. We assume that the cost functional is uniform. If addition of one more zero-complexity group requires costs, then the cost functional cannot be correct. If upon increasing complexity of any of the organized groups the cost of organization can decrease, then the cost functional needs not be monotone. Definitions 8–10 suggest the following variants of the cost functional:

$$P(C(g_1),\dots,C(g_k)) = [C(g_1)+\dots+C(g_k)-\max(C(g_1),\dots,C(g_k))]^\beta; \quad (1)$$

$$P(C(g_1),\dots,C(g_k)) = [C(g_1)+\dots+C(g_k)]^\beta; \quad (2)$$

$$P(C(g_1),\dots,C(g_k),C(g)) = C(g)/\max(C(g_1),\dots,C(g_k))-1; \quad (3)$$

$$P(C(g_1),\dots,C(g_k),C(g)) = \sum_{i=\overline{1,k}} C(g)-C(g_i), \quad (4)$$

where the group $g = g_1 \cup \dots \cup g_k$ is organized from the subgroups $g_1,\dots,g_k$, $\beta \in (0;+\infty)$.

Functional (1) is uniform, correct, and monotone. The cost of organization is defined by the sum of complexities of groups organized beginning from the most complex one.

Functional (2) is uniform and monotone, but not correct.

Functional (3) is uniform and correct, but not monotone. The relative indicator, cost of organization, is defined by the ratio of group complexity to the maximal complexity of the subgroup. We assume that $P(0,\dots,0,0) = 0$.

Functional (4) is uniform, but not correct and monotone. The absolute indicator, cost of organization, is defined by the difference between the group complexity and the complexities of subgroups.

4. KINDS OF OPTIMAL ORGANIZATION FOR DIFFERENT COST FUNCTIONALS

In what follows, we make use of the inequality which is readily proved by induction on n:

$$(x_1+\dots+x_n)^y \geq x_1^y+\dots+x_n^y \quad \text{for any} \quad x_1\geq 0,\dots,x_n \geq 0 \quad \text{for} \quad y \geq 1. \quad (5)$$

Assertion 1. For $\beta \geq 1$, solution of the problem of optimal organization of the groups $f_1,\dots,f_m$ with the cost functional (1) exists in the class of 2-organizations.

Proof. Let $G = (V, E)$ be an optimal organization. We consider $g \in V \backslash \{a_1\},\dots,\{a_n\}$, $Q(g) = \{g_1,\dots,g_k\}$, $k \geq 3$, $C(g_i) = C_i$, $C_1 = \max(C_1,\dots,C_k)$. If there is no such vertex, then G already is the 2-organization (see Definition 4). Otherwise, we construct the organization C' by removing the edges going from the vertices $g_1,\dots,g_k$ to g, adding vertices and edges $h_2 = g_1 \cup g_2$, $Q'(h_2) = \{g_1, g_2\}$; $h_3 = h_2 \cup g_3$, $Q'(h_3) = \{h_2, g_3\}$, and so on, $h_{k-1} = h_{k-2} \cup g_{k-1}$, $Q'(h_{k-1}) = \{h_{k-2}, g_{k-1}\}$. Then g is organized from the vertices $Q'(g) = \{h_{k-1}, g_k\}$. Therefore, simultaneous organization of $g_1,\dots,g_k$ in g was replaced by sequential additions $g_2,\dots,g_k$. In doing so, two edges enter all the added vertices, g also is organized of two subgroups. The number of vertices g with $|Q(g)| \geq 3$ in the resulting graph G' was reduced by one. If we prove that G' is optimal, then continuation of this operation will provide the desired optimal 2-organization.

The label $R(g)$ of the vertex g was included in $P(G)$; we denote it by $P_1 = (C_2+\dots+C_k)^\beta$. New labels $P_2 = R'(h_2)+\dots+R'(h_{k-1})+R'(g) = C_2^\beta+\dots+C_k^\beta$ will be included in $P(G')$ instead of P_1. Since $\beta \geq 1$, it follows from (5) that $P_2 \leq P_1$. Consequently, $P(G') \leq P(G)$, which proves the assertion.

Theorem 1. *For $\beta \geq 1$ and $\alpha\beta \geq 1$, solution of the problem of optimal organization of the groups $f_1,\dots,f_m$ with the cost functional* (1) *exists in the class of sequential organizations.*

Proof. Let $G = (V, E)$ be the optimal 2-organization (see Assertion 1). A vertex of the graph G will be called incorrect if it is organized of two nonelementary groups; otherwise, it will be called correct. We call the vertex h the successor of g if $g \neq h$ and there exists a path from h to g.

If there are no incorrect vertices in G, then G is the sequential organization (see Definition 3). Otherwise, we construct the optimal 2-organization G' where the number of incorrect vertices is

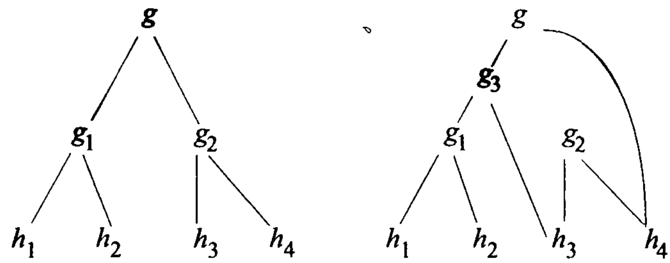

Fig. 1. Reconstruction of the organization graph. Upward orientation of the edges.

one less than in G. Then, by repeating this operation as many times as required we get the optimal sequential organization.

Let g be an incorrect vertex such that all its successors are correct, $Q(g) = \{g_1, g_2\}$. Let in turn $Q(g_1) = \{h_1, h_2\}$, $Q(g_2) = \{h_3, h_4\}$. We assume without loss of generality that $C(g_1) \geq C(g_2)$, $C(h_1) = C_1 \geq C_2 = C(h_2)$, $C_3 = C(g_3)$, $C_4 = C(g_4)$, h_4 is elementary. Since g_1 is a correct vertex, at least one of the vertices h_1 or h_2 is elementary. G is loop-free (see Definition 1b). Consequently, $h_1 \cap h_2 = \varnothing$, similarly, $h_3 \cap h_4 = \varnothing$. Then, $C(g_1) = (C_1^{1/\alpha} + C_2^{1/\alpha})^\alpha$, $C(g_2) = (C_3^{1/\alpha} + C_4^{1/\alpha})^\alpha$.

We reconstruct G by adding the vertex $g_3 = g_1 \cup h_3$ and changing the edges so that $Q(g) = \{g_3, h_4\}$, $Q(g_3) = \{g_1, h_3\}$. The corresponding part of G before and after reconstruction is shown in Fig. 1 to the left and right, correspondingly.

If one edge goes from g_2 in G and g_2 is not contained among $f_1, \ldots, f_m$, then we eliminate g_2. We denote the resulting 2-organization of the groups $f_1, \ldots, f_m$ by G_1. The cost of G includes

$$P_1 = R(g_1) + R(g_2) + R(g) = R(g_1) + R(g_2) + \left(C_3^{1/\alpha} + C_4^{1/\alpha}\right)^{\alpha\beta}.$$

If the vertex g_2 was retained, then instead of P_1 the cost of G_1 includes

$$P_2 = R'(g_1) + R'(g_2) + R(g_3) + R'(g) = R(g_1) + R(g_2) + C_3^\beta + C_4^\beta.$$

With regard for (5), for $\alpha\beta \geq 1$ we obtain $(C_3^{1/\alpha} + C_4^{1/\alpha})^{\alpha\beta} \geq (C_3^{1/\alpha})^{\alpha\beta} + (C_4^{1/\alpha})^{\alpha\beta}$, whence it follows that $P_2 \leq P_1$. Consequently, $P(G_1) \leq P(G)$. Removal of g_2 can only reduce P_2.

Therefore, G_1 is optimal. If g_3 is correct, then we get the desired graph G'. Otherwise, we reason as follows. The level of g_3 is lower that of g, all successor vertices of g_3 are correct. We repeat the above reconstruction by taking G_1 as G and the vertex g_3 as g. Therefore, we again reduce the level of the incorrect vertex. By repeating these actions, either obtain the desired graph G' at the current step or reach the instant where the level of g_3 is two. In this case, g_3 is correct, which is what we set out to prove.

Assertion 2. *Solution of the problem of optimal organization of the group* $f_1, \ldots, f_m$ *with the cost functional* (3) *exists in the class of 2-organizations.*

Proof is repeats that of Assertion 1. By retaining the notation of this proof, we assume that $C_i = C(h_i)$, $i = \overline{1,k}$, $h_1 = g_1$, $h_k = g$. Then, $C(g_i) \leq C_1 \leq C_2 \leq \ldots \leq C_k$ is satisfied for $i = \overline{1,k}$; it is possible to set down P_1, P_2: $P_1 = C_k/C_1 - 1$, $P_2 = C_2/C_1 + C_3/C_2 + \ldots + C_k/C_{k-1} - (k-1)$.

We prove by induction on k that $P_2 \leq P_1$. We denote P_1, P_2 for each k by $P_1(k)$, $P_2(k)$. We get $P_2(2) = P_1(2)$. Let $P_2(k) \leq P_1(k)$ be satisfied for $k \leq j$. We assume that $k = j+1$:

$$P_1(j+1) = C_{j+1}/C_1 - 1 = (C_{j+1} - C_j + C_j)/C_1 - 1 = P_1(j) + (C_{j+1} - C_j)/C_1,$$
$$P_2(j+1) = C_2/C_1 + C_3/C_2 + \ldots + C_j/C_{j-1} + C_{j+1}/C_j - j = P_2(j) + (C_{j+1} - C_j)/C_j.$$

Since $C_j \geq C_1$ and $P_2(j) \leq P_1(j)$ by assumption, we get that $P_2(j+1) \leq P_1(j+1)$, that is, $P_2 \leq P_1$, which proves the assertion.

Theorem 2. *Solution of the problem of optimal organization of the groups $f_1, \ldots, f_m$ with the cost functional* (3) *exists in the class of sequential organizations.*

Proof follows that of Theorem 1, the notation is the same, the optimal 2-organization exists by virtue of Assumption 2. The values of P_1 and P_2 are representable as $P_1 = R(g_1) + R(g_2) + C(g)/C(g_1) - 1$ and $P_2 = R(g_1) + R(g_2) + C(g_3)/C(g_1) - 1 + C(g)/C(g_3) - 1$. We denote $x = C(g)$, $y = C(g_3)$, and $z = C(g_1)$. Then, $z \le y \le x$ is satisfied, and it is possible to set down

$$P_1 - P_2 = x/z - 1 - y/z + 1 - x/y + 1 = (xy + yz - y^2 - xz)/yz.$$

We denote $\xi(x) = xy + yz - y^2 - xz$ and differentiate $\xi'(x) = y - z \ge 0$ by virtue of $y \ge z$. Further, $\xi(y) = y^2 + yz - y^2 - yz = 0$; consequently, $\xi(x) \ge 0$ is satisfied for all $x \ge y$, that is, $P_2 \le P_1$, which proves the theorem.

5. PROBLEM OF OPTIMAL SEQUENTIAL ORGANIZATION. ALGORITHMS OF SOLUTION

Definition 11. The sequential organization of the group $f_1, \ldots, f_m$ having the minimal cost among all such organizations will be called the optimal sequential organization. The problem of seeking one of such organizations will be called the problem of optimal sequential organization.

Definition 12. *The oriented graph* $H = (V_H, E_H) : V_H = F \cup \{\varnothing\}$, $E_H = E'_H \cup E''_H \cup E'''_H$, *where* $E'_H = \{(\varnothing, \{a_i\}) : i = \overline{1, n}\}$, $E''_H = \{(\{a_i\}, \{a_i, a_j\}) : 1 \le i < j \le n\}$, $E'''_H = \{(g, g \cup \{a_i\}) : g \in V,\ |g| \ge 2,\ a_i \notin g,\ i = \overline{1, n}\}$ *(see Fig. 2), will be called the problem graph. We define by* $\lambda : E_H \to R^+$ *the weight of graph edges and assume that the edges from* E'_H *have zero weight. Any edge from* $E''_H \cup E'''_H$ *has the form* $e = \{g, g \cup \{a_i\}\}$ *and corresponds to the organization of* g *and* $\{a_i\}$ *in* $g \cup \{a_i\}$. *We assume that* $\lambda(e) = P(C(g), C(\{a_i\}), C(g \cup \{a_i\}))$.

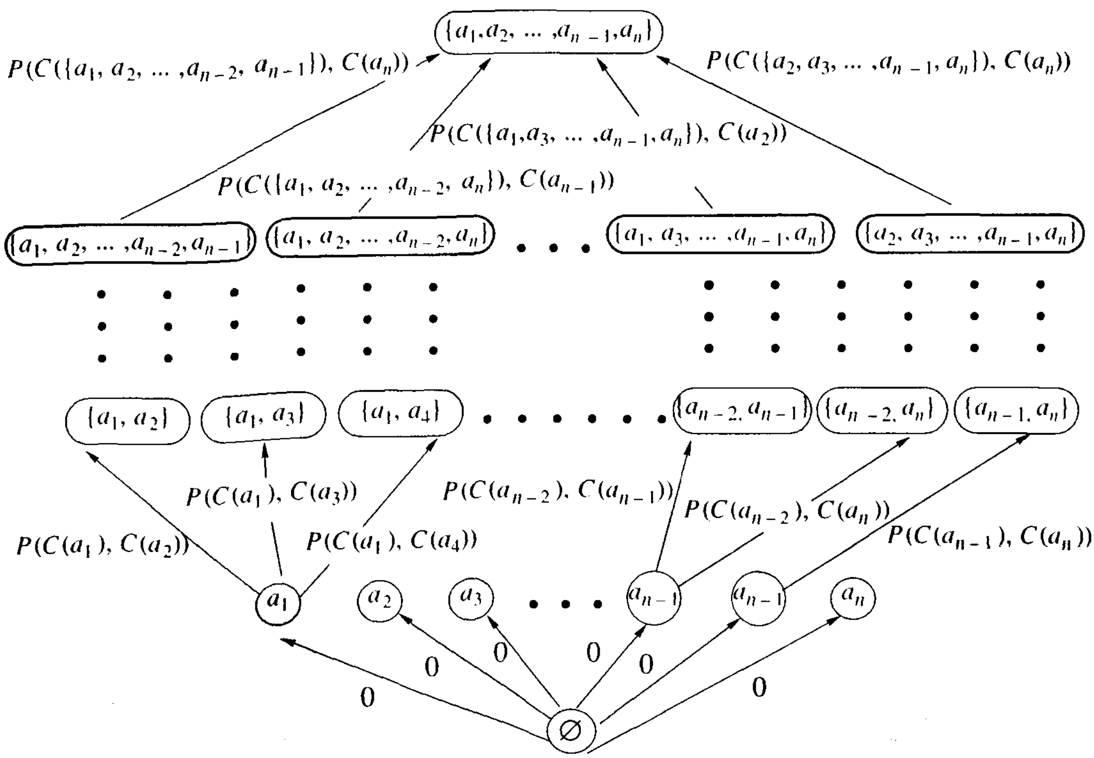

Fig. 2. Graph $H = (V_H, E_H)$ of the problem of sequential organization.

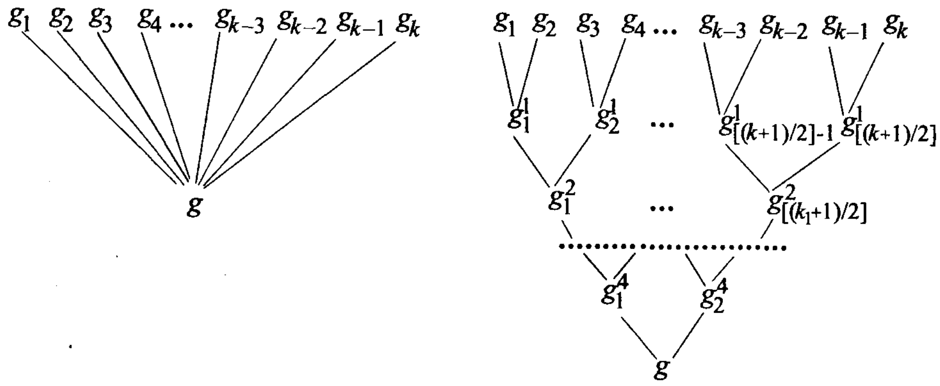

Fig. 3. Reduction to the normalized graph of the problem.

Figure 2 depicts the problem graph. To simplify the figure, the last argument in P and the braces of the elementary groups were omitted.

Definition 13. *Let us consider the subtree D of the problem graph H rooted in $\varnothing$ which comprises $f_1, \dots, f_m$ whose leaves are contained among $f_1, \dots, f_m$. The problem of seeking such a minimal-weight subtree will be called the problem of optimal subtree in H. By the weight $\lambda(D)$ of the subtree D is meant the sum of weights of edges D.*

In what follows, by the subtree is meant the above subtree, unless otherwise specified.

Theorem 3. *The problem of optimal sequential organization is equivalent to that of optimal subtree in H.*

Proof. Let $G = (V, E)$ be a sequential organization of the groups $f_1, \dots, f_m$. Let us construct the subtree $D = (V_D, E_D)$ of the graph H, the set of vertices $V_D = V'_D \cup V''_D \cup \{\varnothing\}$, where $V'_D = \{g \in V, |g| \geq 2\}$, $V''_D = \{\{a_i\} : \exists j > i, \{a_i, a_j\} \in V\}$. We consider $g \in V$, $|g| \geq 3$; then, in G $Q(g) = \{h, \{a_i\}\}$, g, $h \in V_D$, include the edge $e = (h, g)$ in E_D, and obtain here $R(g) = \lambda(e)$. We consider $g = \{a_i, a_j\} \in V$, $i < j$, then $g, \{a_i\} \in V_D$. We include the edge $e = (\{a_i\}, g)$ in E_D. Again, we get $R(g) = \lambda(e)$. For $\{a_i\} \in V_D$, we include the edge $e = (\varnothing, \{a_i\})$ in E_D, $\lambda(e) = 0$. By construction, D is a subtree, $P(G) = \lambda(D)$.

Inversely, let $D = (V_D, E_D)$ be a subtree of H. We construct the sequential organization $G = (V, E)$ of the group $f_1, \dots, f_m$: $V = (V_D \cup \{\{a_1\}, \dots, \{a_n\}\}) \backslash \{\varnothing\}$. One edge $e = (h, g)$ enters the vertex $g \in V$, $|g| \geq 2$ D; we add the edges (h, g), $(g \backslash h, g)$ to E, then $R(g) = \lambda(e)$. By construction, $P(G) = \lambda(D)$.

Therefore, a sequential organization G, $P(G) = \lambda(D)$, corresponds to each subtree D and *vice versa*, which proves the theorem.

Definition 14. Let us consider $H = (V_H, E_H)$. We denote for $g \in V_H$ by $S(g)$ the set of vertices to which edges from g come. Let $g \in V_H$, $k = |S(g)| \geq 3$, $S(g) = \{g_1, \dots, g_k\}$. We transform H by removing the edges (g, g_i), $i = \overline{1, k}$, and adding $k_1 = \lfloor (k+1)/2 \rfloor$ vertices denoted by $g_1^1, \dots, g_{k_1}^1$ and edges $(g^1_{\lfloor (i+1)/2 \rfloor}, g_i)$, $i = \overline{1, k}$, having weight equal to the weight of the removed edges (g, g_i). If $k_1 \geq 3$, we add $k_2 = \lfloor (k_1+1)/2 \rfloor$ vertices denoted by $g_1^2, \dots, g_{k_2}^2$ and edges $(g^2_{\lfloor (i+1)/2 \rfloor}, g_i^1)$, $i = \overline{1, k_1}$, of zero weight, and so on. At the current step we add two vertices g_1^q and g_2^q and edges (g, g_1^q) and (g, g_2^q) of zero weight. By performing these steps for all $g \in V_H$, $|S(g)| \geq 3$, we obtain the graph $N = (V_H, E_N)$ from each of whose vertices at most two edges go out. It will be called the normalized graph of the problem.

The above reconstruction for $k = 2^{q+1}$ is shown in Fig. 3.

Assertion 3. *The problem of optimal subtree in H is equivalent to that of the optimal subtree on the normalized problem graph N.*

Proof. Let us consider the subtree $D = (V_D, E_D)$ of the graph $H = (V_H, E_H)$. We construct the subtree $D' = (V'_D, E'_D)$ of the graph $N = (V_N, E_N)$. We add to V'_D all vertices from V_D. For $g \in V_D$, $|S(g)| \leq 2$, we add the edges outgoing from g in E'_D. We consider $g \in V_D$, $S(g) = \{g_1, \ldots, g_k\}$, $k \geq 3$. Then, there are in N the vertices $g^j_1, \ldots, g^j_{k_j}$, $j = \overline{1, q}$ (see Definition 14). Let ℓ edges go out in D from g to $g_{i_1}, \ldots, g_{i_\ell}$; we add to V'_D the vertices $g^1_{\lfloor (i_j+1)/2 \rfloor}$ and to E'_D the edges $(g^1_{\lfloor (i_j+1)/2}, g_{i_j})$, $j = \overline{1, \ell}$. For the added vertices, we add the edges leading to them in the graph N and the corresponding vertices, and so on until the edges going out of g are added. We obtain the subtree D' of the graph N. By construction, $\lambda(D') = \lambda(D)$.

Inversely, let us consider the subtree $D' = (V'_D, E'_D)$ of the graph N. We construct the subtree $D = (V_D, E_D)$ of the graph H. We add to V_D all vertices from $V'_D \cap V_H$. For $g \in V'_D \cap V_H$, $|S(g)| \leq 2$, we add to D the edges going out of g to D'. For $g \in V'_D \cap V_H$, $Q(g) = \{g_1, \ldots, g_k\}$, $k \geq 3$, some $0 \leq \ell \leq k$ vertices $g_{i_1}, \ldots, g_{i_\ell}$ (see Definition 14) belong to V'_D. Then, we add to E_D the edges (g, g_{i_j}), $j = \overline{1, \ell}$, and obtain the subtree D of the graph H; at that, $\lambda(D') = \lambda(D)$ by construction. Therefore, to each subtree of N a subtree of H of the same weight corresponds and *vice versa*, which proves the assertion.

Theorem 4. *There exists an algorithm solving the problem of optimal subtree in $N = (V_N, E_N)$ by comparing less than $V_2(N)3^m$ weights of different subtrees, where $V_2(N) = |\{g \in V_N : |S(g)| = 2\}|$. Constriction of the algorithm is given in the proof.*

Proof. We denote $M = \{f_1, \ldots, f_m\}$, $L = 2^M \backslash \{\varnothing\}$ and consider $v \in L$ and $g \in V_N$. We denote by $\lambda(g, v)$ the minimal weight of the subtree with root at g which contains vertices from v and whose leaves are contained among the vertices of v. If there is no such tree, then we assume that $\lambda(g, v) = \infty$.

Let $|S(g)| = 0$: if $g \notin M$, then $\lambda(g, v) = \infty$ for any $v \in L$; if $g \in M$, then $\lambda(g, \{g\}) = 0$, $\lambda(g, v) = \infty$ for any $v \neq \{g\}$.

Let one edge $e = (g, h)$ go out of g. If $g \notin M$, then for any $v \in L$ we obtain $\lambda(g, v) = \lambda(h, v) + \lambda(e)$, and the subtree corresponding to $\lambda(g, v)$ is constructed as a union of the edge e and the subtree for $\lambda(h, v)$. If $g \in M$: for any $v \in L$, $g \notin v$ we have $\lambda(g, v) = \lambda(h, v) + \lambda(e)$; for any $v \in L$, $g \in v$, $v \neq \{g\}$ we have $\lambda(g, v) = \lambda(h, v \backslash \{g\}) + \lambda(e)$ because g is already at the tree root. If $g \in M$, then $\lambda(g, \{g\}) = 0$.

Let two edges $e_1 = (g, h_1)$ and $e_2 = (g, h_2)$ go out of g. Let us consider $v \in L$; in the subtree corresponding to $\lambda(g, v)$ some collection $v_1 \subseteq v$ is contained in the subtree with root at h_1; collection $v_2 = v \backslash v_1$ (or $v_2 = v_1 \backslash (v_1 \cup \{g\})$ if $g \in M$) is contained in the subtree with root at h_2. If $v_1 \neq \varnothing$, $v_2 \neq \varnothing$, then $\lambda(g, v) = \lambda(h_1, v_1) + \lambda(e_1) + \lambda(h_2, v_2) + \lambda(e_2)$. If $v_1 \varnothing$, $v_2 = \varnothing$, then $\lambda(g, v) = \lambda(h_2, v_2) + \lambda(e_2)$. If $v_1 \neq \varnothing$, $v_2 = \varnothing$, then $\lambda(g, v) = \lambda(h_1, v_1) + \lambda(e_1)$. If $g \in M$, then $\lambda(g, \{g\}) = 0$. By comparing at most $2^{|v|}$ variants of decomposition of v on v_1, v_2, we determine $\lambda(g, v)$. For all $v \in L$, we compare at most $\sum\limits_{i=\overline{1,m}} C^i_m 2^i < 3^m$ variants.

For vertices from which no edge goes out, $\lambda(g, v)$ and the corresponding subtrees are known for any $v \in L$. The problem of subtrees will be said to be solved for such vertices. Owing to acyclicity of N, there will be $g \in V_N$ such that the edges from g go to the vertices for which the problem of subtree is solved. Then, we solve the problem for g, which requires less than 3^m comparisons of the subtree weights. We proceed until the problem is solved for all vertices $g \in V_N$; then $\lambda(\varnothing, M)$ and the corresponding subtrees are the required ones, which proves the theorem.

Corollary 1. *There exists an algorithm solving the problem of optimal sequential organization by comparing less than* $(n+1)2^n 3^m$ *weights of the subtrees of the graph* N.

Proof. For any $g \in V_H$, $k = |S(g)| \geq 3$, we add to N at most $2k$ vertices. From the vertex H of the ith level, $n-i$, $i = \overline{2, n-3}$, edges go out. For all vertices of the ith level, we add at most $2C_n^i(n-i)$ vertices in N. From the vertex $\{a_i\}$, $j = \overline{1, n-1}$, of the graph H, $n-j$ edges go out; for all vertices of the first level we add to N at most $\sum_{j=\overline{1,n-1}} 2(n-j)$ vertices. For $\varnothing$, we add to N at most $2n$ vertices. All in all, at most $\sum_{i=\overline{2,n-3}} 2C_n^i(n-i) + \sum_{j=\overline{1,n-1}} 2(n-j) + 2n$ vertices are added. We estimate the last expression by $n2^n$ taking into account that $iC_n^i = nC_{n-1}^{i-1}$. Since $|V_H| = 2^n$, $V_2(N) \leq |V_N| \leq (n+1)2^n$, which proves the corollary.

Corollary 1 gives the worst-case upper bound. Let us generate in an arbitrary way a collection of groups (each agent is included in a group with probability 0.5), calculate $V_2(N)$, and apply Theorem 2 to estimate the average complexity by 100 tests. For $m, n = 15$, complexity is approximately 3×10^8, that is, is still acceptable.

For $C(a_1) = \ldots = C(a_n) = C$, complexity of the group is defined only by its level. We denote by $P_i = P(i^\alpha C, C, (i+1)^\alpha C)$ the cost of organizing a group of the ith level with the elementary group. The problem of optimal sequential organization is fully defined by the values $P_1, \ldots, P_{n-1}$ and the collection $f_1, \ldots, f_m$.

Theorem 5. *For* $C(a_1) = \ldots = C(a_n)$, *the problem of optimal sequential organization* $f_1, \ldots, f_m$, $|f_i| \leq 3$, $i = \overline{1, m}$, *is* NP*-complete for any* $P_1 > 0$, P_2.

Proof. We prove membership to the NP class. In the optimal sequential organization, f_i is organized in some sequence. By generating this sequence by a nondeterministic machine for each f_i, we remove all repeated groups and calculate the cost in polynomial time.

Let us consider the problem of representatives in the 2-sets: given are the set $X = \{x_1, \ldots, x_n\}$ and two-element subsets $Y_1, \ldots, Y_m \subseteq X$; needed is to determine the minimal-cardinality $Y \subset X$ such that $|Y \cap Y_i| \geq 1$ for $i = \overline{1, m}$. The problem of representatives in the 2-sets is NP-complete (see [6]).

Let $Y_i = \{x_{j_i}, x_{k_i}\}$, $i = \overline{1, m}$, $1 \leq j_i < k_i \leq n$. We assume that $A = \{a_0, a_1, \ldots, a_n\}$, $f_i = \{a_0, a_{j_i}, a_{k_i}\}$. Let the set of representatives $Y = \{x_{\ell_1}, \ldots, x_{\ell_q}\} \subset X$ be given. We construct $G = (V, E)$: $V = \{\{a_0\}, \{a_1\}, \ldots, \{a_n\}, f_1, \ldots, f_m, \{a_0, a_{\ell_1}\}, \ldots, \{a_0, a_{\ell_q}\}\}$. To organize $\{a_0, a_{\ell_1}\}, \ldots, \{a_0, a_{\ell_q}\}$, we add edges to E. For $Y_i = \{x_{j_i}, x_{k_i}\}$, either $x_{j_i} \in Y$ or $x_{k_i} \in Y$ is satisfied; consequently, $\{a_0, a_{j_i}\} \in V$ or $\{a_0, a_{k_i}\} \in V$. We organize f_i by means of one of these groups. As the result, we construct the sequential organization G of the groups $f_1, \ldots, f_m$ of cost $qP_1 + mP_2$.

Inversely, let sequential organization $G = (V, E)$ of the groups $f_1, \ldots, f_m$, $P(G) = qP_1 + mP_2$, where q is the number of groups of level two in G, be given. Let $g = \{a_i, a_j\} \in V$, $1 \leq i < j \leq n$. An edge can go out of g only to $f = \{a_0, a_i, a_j\}$. We remove g and organize f from $\{a_0, a_i\}$ and $\{a_j\}$ by adding $\{a_0, a_i\}$, if necessary. By continuing in the same way, we determine the sequential organization $G' = (V', E')$ comprising q' groups $\{a_0, a_{\ell_1}\}, \ldots, \{a_0, a_{\ell_{q'}}\}$ of level two, $P(G') = q'P_1 + mP_2$, $q' \leq q$. Let us then consider $Y = \{x_{\ell_1}, \ldots, x_{\ell_{q'}}\} \subset X$. For $f_i = \{a_0, a_{j_i}, a_{k_i}\}$, either $\{a_0, a_{j_i}\} \in V'$ or $\{a_0, a_{k_i}\} \in V'$ is satisfied, that is, there exists $1 \leq r \leq q'$: $a_{j_i} = a_{\ell_r}$ or $a_{k_i} = a_{\ell_r}$; consequently, $x_{j_i} = x_{\ell_r}$ or $x_{k_i} = x_{\ell_r}$, $|Y_i \cap Y| \geq 1$. So, Y is the set of representatives of q' elements.

Let us determine the optimal sequential organization G, $P(G) = qP_1 + mP_2$; then, in a polynomial time we can determine the set of representatives Y of q' elements, $q' \leq q$. If there existed a set of representatives of $q'' < q' \leq q$ elements, there would be a sequential organization G'', $P(G'') = q''P_1 + mP_2 < P(G)$, which contradicts optimality of G. Consequently, Y is the solution of the problem of representatives in 2-sets, which proves the theorem.

Definition 15. The groups of the set $U = \{f_{i_1} \cap \ldots \cap f_{i_k} : 1 \leq k \leq m,\ 1 \leq i_1 < \ldots < i_k \leq m\} \setminus \{\{a_1\}, \ldots, \{a_n\}, \varnothing\}$ will be called the nodal groups.

Assertion 4. *For $C(a_1) = \ldots = C(a_n)$, there exists the optimal sequential organization $G = (V, E)$ for which any $g \in V$ from which more than one edge goes out is either nodal or elementary.*

Proof. Let $G = (V, E)$ be the optimal sequential organization of the groups $f_1, \ldots, f_m$. Let at least two edges go out from $g \in V \setminus \{\{a_1\}, \ldots, \{a_n\}\}$, $g \notin U$, in h_1, ℓ_1, there existing no path from g to any vertex with the same properties. Let $h_1 - h_2 - \ldots - h_{n_1}$ be the path from h_1 to $h_{n_1} \in U$ which comprises no other nodal groups; similarly, $\ell_1 - \ell_2 - \ldots - \ell_{n_2}$ is the path from ℓ_1 to $\ell_{n_2} \in U$. By construction, precisely one edge goes out of each vertex h_i, $i = \overline{1, n_1 - 1}$, ℓ_j, $j = \overline{1, n_2 - 1}$.

Let us consider $g' = h_{n_1} \cap \ell_{n_2}$. We have $g \subseteq g'$; consequently, g' is nonelementary and $g' \in U$. Then, $g \subset g'$ because $g \notin U$. The vertices $f_1, \ldots, f_m$ are not included among $h_1, \ldots, h_{n_1 - 1}$, $\ell_1, \ldots, \ell_{n_2 - 1}$, and we remove them. Let $g' \setminus g = \{a_{i_1}, \ldots, a_{i_k}\}$; then we add to G the vertices $g_j = g \cup \{a_{i_1}, \ldots, a_{i_j}\}$, $j = \overline{1, k}$, and edges (g_{j-1}, g_j), $(\{a_{i_j}\}, g_j)$, where $g_0 = g$. Similarly, we complete sequentially construction of g' to h_{n_1} and ℓ_{n_2}. As the result, we get a sequential organization G' of the groups $f_1, \ldots, f_m$.

Instead of the labels of the vertices $h_1, \ldots, h_k$, $\ell_1, \ldots, \ell_k$, $P(G')$ includes the labels of vertices $g_1, \ldots, g_k$, that is, $P(G') \leq P(G)$. We did not add to G' any nonnodal vertex from which more than one edge would go out. From g in G' one less edge goes out that in G. We continue the above actions until one edge goes out of g.

As the result, we construct the optimal sequential organization of the groups $f_1, \ldots, f_m$ where the number of vertices $g \notin U \cup \{\{a_1\}, \ldots, \{a_n\}\}$ with more than one outgoing edge is one less than in G. By continuing these actions, we obtain the desired optimal sequential organization, which proves the assertion.

Assertion 5. *There exists an optimal sequential organization $G = (V, E)$ for which the following is satisfied in addition to the conditions of Assertion 4. If g, $h \in V \cap U$, $h \subset g$, and there exists a path from h to g containing no nodal vertices, then there does not exist $g_3 \in U$, $g_1 \subset g_3 \subset g_2$. If there is no path to g from any nodal vertex of G, then there exists no $h \in U$, $h \subset g$.*

Proof. Let us consider the optimal sequential organization of G for which the conditions of Assertion 4 are satisfied, and some path $h_1 - \ldots - h_k$ in G, $k \geq 2$ $h_k \in U$, $h_1 \in U \cup \{\{a_1\}, \ldots, \{a_n\}\}$, $h_2, \ldots, h_{k-1} \notin U$. The path will be called incorrect if there exists $g' \in U$, $h_1 \subset g' \subset h_k$.

Let $h_1 - \ldots - h_k$ be an incorrect path, then $g' \in U$ corresponds to it. We remove the vertices $h_2, \ldots, h_{k-1}$ from G. Let $g' \setminus h_1 = \{a_{i_1}, \ldots, a_{i_r}\}$; we add to G the vertices $h'_j = h_1 \cup \{a_{i_1}, \ldots, a_{i_j}\}$, $j = \overline{1, r}$, by organizing h'_j from h'_{j-1} and $\{a_{i_j}\}$, where $h'_0 = h_1$. We complete similarly construction of g' to h_k and obtain a new organization of G' with $P(G') = P(G)$. If the path from h_1 to g' or from g' to h_k is incorrect, then we perform this procedure for these subpaths, and so on until we get the optimal organization where any subpath from h_1 to h_k is correct. Consequently, the number of incorrect paths was decreased by one as compared with G. By repeating these actions, we obtain the optimal sequential organization all of whose paths are correct, which proves the assertion.

For further presentation, we redefine the problem graph $H = (V_H, E_H)$ for $C(a_1) = \ldots = C(a_n)$. The set of vertices $V_H = U \cup \{\varnothing\}$. We consider g_1, $g_2 \in V_H$, $g_1 \subset g_2$; there exists no $g_3 \in V_H$ for which $g_1 \subset g_3 \subset g_2$. For each such pair, we include in E_H the edge $e = (g_1, g_2)$ by assuming that

$\lambda(e) = P_{|g_1|} + P_{|g_1|+1} + \ldots + P_{|g_2|-1}$, where $P_0 = 0$, that is, the weight $\lambda(e)$ is equal to the cost of sequential completion of construction of g_1 to g_2.

Theorem 6. *For $C(a_1) = \ldots = C(a_n)$, the problem of optimal sequential organization is equivalent to that of the optimal subtree in H.*

Proof. Let $G = (V, E)$ be the optimal sequential organization $f_1, \ldots, f_m$ satisfying the conditions of Assertion 5. Let us construct a subtree $D = (V_D, E_D)$ of the graph H. The set of vertices $V_D = V_U \cup \{\varnothing\}$, where $V_U \subset V$ is the set of nodal groups of V. Let us consider $g \in V_U$; let $Q(g) = \{g_1, \{a_{i_1}\}\}$. If $g_1 \notin V_U \cup \{\{a_1\}, \ldots, \{a_n\}\}$, then we consider $Q(g_1) = \{g_2, \{a_{i_2}\}\}$, repeat the arguments for g_2 and so on until we reach $g_k \in V_U \cup \{\{a_1\}, \ldots, \{a_n\}\}$. Either g_k is elementary, then we add the edge $(\varnothing, g)$ to D, or $g_k \in V_U$, and then we add to D the edge (g_k, g). In the first case, there exists no $g' \in U$, $g' \subset g$, and in the second case there exists no $g' \in U$, $g_k \subset g' \subset g$, that is, in both cases the added edge belongs to E_H, its weight being equal to the total cost of vertex labels $g, g_1, \ldots, g_{k-1}$. Each vertex of D, except for $\varnothing$, includes precisely one edge from a lower-level vertex; consequently, D is a subtree of H. The label of each nonelementary group G is included in $\lambda(D)$ precisely once; consequently, $P(G) = \lambda(D)$.

Inversely, let $D = (V_D, E_D)$ be a subtree of the graph H. We construct a sequential organization $G = (V, E)$ and add to V the vertices $(V_D \cup \{\{a_1\}, \ldots, \{a_n\}\}) \backslash \{\varnothing\}$. Let $g \in V_D$; on the tree D, precisely one edge $e = (h, g)$ comes to it. Let $g \backslash h = \{a_{i_1}, \ldots, a_{i_k}\}$; then, we add to V the vertices $h_j = h \cup \{a_{i_1}, \ldots, a_{i_j}\}$, except for the elementary $\{a_{i_1}\}$ vertex $h = \varnothing$, $j = \overline{1, k-1}$, by organizing h_j from h_{j-1} and $\{a_{i_j}\}$, where $h_0 = h$. The sum of labels of g and the vertices added to V will be $\lambda(e)$. As the result, we construct G, $P(G) = \lambda(D)$, which proves the theorem.

In the case of $C(a_1) = \ldots = C(a_n)$, the algorithm to solve the problem of optimal subtree in H coincides completely with the general algorithm, which one can ascertain by repeating word for word Assertion 3, and Theorem 4.

Corollary 2 (of Theorem 4). *For $C(a_1) = \ldots = C(a_n)$, there exists an algorithm solving the problem of optimal sequential organization which compares less than $22^{2m}3^m$ weights of different subtrees of the normalized graph of the problem N.*

Proof. For $C(a_1) = \ldots = C(a_n)$, the graph H has at most 2^m vertices from each of which at most $2^m - 1$ edges go out. Consequently, when constructing the graph N we add at most $2^{m+1} - 2$ vertices for each vertex from H. All in all, we add at most $2^{2m+1} - 2^{m+1}$ vertices. Therefore, $V_2(N) \leq |V_N| < 22^{2m}$, which proves the corollary.

To construct H, one has to determine at most 2^m vertices and at most 2^{2m-1} edges. To determine a vertex, we perform at most mn operations, and at most n operations to determine an edge. The order of complexity of the remaining operations (construction of N, passage from the subtree in N to the subtree in H) does not exceed 2^{2m}. Therefore, the order of complexity of the algorithm to solve the problem for $C(a_1) = \ldots = C(a_n)$ does not exceed $22^{2m}3^m + n(2^{2m-1} + m2^m)$. Only the second term depends on n and, at that, depends linearly, which enables one to solve the problem for greater n. By generating randomly a collection of groups (the agent is included in a group with the probability 0.5) and estimating complexity on the average by 100 tests, we obtain that the problem is solvable if m does not exceed fifteen.

6. CONCLUSIONS

If the environment does not vary (static case), then one can minimize the costs and create an optimal-structure system by solving the problem of optimal organization. For the functional (1) for $\beta \geq 1$, $\alpha\beta \geq 1$, and the functional (3), the problem of optimal organization can be solved using the above algorithms to determine the optimal sequential organization (see Theorems 1 and 2). For the functional (4), there exists an optimal 2-organization, the sequential organization in the general case does not exist (see [7]). For the monotone cost functionals, existence of the optimal tree of organization of one group was proved (see [7]). It can be determined using the algorithms of [7]. By using the form of a group which is optimal among the sequential organizations (see [7]) and relying on Theorems 1 and 2, one can establish the optimal organization of one group for the functionals (1) and (3). Besides the domain $\alpha < 1$, $\beta > 1$, for the functional (2), the optimal organization of one group was determined in [7].

If the environment varies (dynamic case), then the collection of groups $f_1, \ldots, f_m$, the set of agents, the cost functional, and complexity parameters can vary. If the cost of passing from one structure to another is given, then one can compare different strategies of reorganization—for example, by the mean result of system operation over a certain time interval. The following strategies can be suggested. The strategy of maximal changes—for each change of the environment, we pass to the optimal organization. The strategy of minimal changes—upon adding new groups and agents, we pass from the previous organization G to an organization which is optimal in G (see Definition 7) and then remove the "excessive" parts of the graph. The majority of the algorithms described in this paper and in [7] can be modified to establish a relatively optimal organization. The strategy of retaining the minimal-complexity structure (see Definition 6)—all the time we retain the simultaneous organization (see Definition 5) requiring the least reconstructions upon changes. By modeling the system behavior, one can establish which of the strategies is preferable under the current conditions, for example, under the given intensity of environmental changes. Verification of the regularity observed in practice is one of the problems of modeling: for rigid (intensive) external changes, it is advantageous to maintain the simple system structure which is complicated as the external actions become easier.

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