DESIGN OF A COORDINATED MANUFACTURING STRUCTURE

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We consider the problem of designing a coordinated manufacturing structure. A method of solution is proposed. Conditions are found for the problem to be epicoordinated and a corresponding proposition is proved. It is shown that the models and methods of the aggregation problem [1] also apply to the design problem of coordinated manufacturing structures. An example is solved.

1. Introduction

A manufacturing structure (MS) is a collection of manufacturing subdivisions (MSD), including their composition and the interrelationships between them. Most approaches to MS optimization rely on "top-down" description of the MS, from plant to shop, from shop to line, etc. [2, 3]. The authors use economic or technological criteria which generally express only the preferences of central management. We refer to central management as the headquarters [4]. But the MS is a collection of MSDs, where the local managers seek to optimize the MS of their unit, which does not necessarily ensure optimality of the overall MS. Below we consider the problem of coordinating the headquarters preferences with the preferences of the managers of groups of MSDs (e.g., shop foremen). These groups are called divisions. The set of MSDs subordinated to one divisions. The set of MSDs subordinated to one division is called a manufacturing block (MB). As an example, consider a MS which is optimal from the headquarters' viewpoint. There are no conflicts of interest if the MSs of different MBs are also optimal from the divisional viewpoint (the case of epicoordination) [4]. But, in practice, the divisions often operate under a variety of conditions which are not optimal from the divisional point of view. The divisional payoff is usually the same (e.g., the salary of the shop foremen). In many cases, the payoff of overloaded divisions is lower than the payoff of divisions which operate under easier conditions. The difference in payoff is attributable to the fact that deviations from plan are more likely in overloaded divisions, while the manufacturing bonus is usually determined by the closeness of actual performance to plan. These cases indicate that a MS optimized without regard to the divisional preferences will not produce optimal operation of the entire manufacturing enterprise.

In this article, we propose an approach to MS design which attempts to coordinate the preferences of the headquarters and the divisions.

2. Substantive Interpretation of the Problem

Consider a manufacturing system consisting of headquarters, n divisions, and m MSDs ($n < m$). The headquarters decides on the MS design by optimal assignments of the MSDs to the divisions. Each division attempts to acquire tightly linked and efficiently operating MSDs. In this way, the interdependences between different MBs are minimized and their operating conditions are improved. The divisional problem thus can be stated in the following term: the division seeks to maximize its own divisional payoff by choosing "good" MSDs subject to local technology constraints, e.g., the total area needed to accommodate the MSDs should not exceed the total manufacturing area allocated to the division by the headquarters, etc.

The headquarters uses the information about the solutions of the divisional problems to identify sets of coordinated MSDs. The problem of designing a coordinated MS with complete information at the headquarters [4] is thus stated in the following form: the headquarters seeks to design a MS by assigning to each division various MSDs from a set of coordinated MSDs so as to minimize the headquarters functional subject to the set of local technology constraints (the constraints on the divisional states) and the following global constraints: 1) each MSD should be assigned to one division only; 2) only those MSDs may be

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TABLE 1

Functional	Notation used in the source reference	Reference
$J_1=\sum_{i=1}^{n}\frac{1}{N_i}\sum_{\substack{j,l\in B_i\\ j\neq l}}\alpha_{jl}$	B – the set of aggregated groups $B=\{B_i\}$, N_i – number of objects in group B_i, α_{jl} – "association strength" between the pair (j, l); number of aggregated groups	[9]
$J_2=\sum_{i=1}^{n}\sum_{\substack{j,l\in B_i\\ j\neq l}}(\alpha_{jl}-\alpha_i)^2$	$\alpha_i=\frac{1}{N_i^2}\sum\alpha_{jl}$ over $j,l\in B_i,\ j\neq l$, α_i – a number closest to all the associations from B_i	[9]
$J_3=\sum_{i=1}^{n}\sum_{\substack{j,l\in B_i\\ j\neq l}}\alpha_{jl}$		[1]
$J_4=\sum_{i=1}^{n}\sum_{\substack{j,l\in B_i\\ j\neq l}}(\alpha_{jl}-\alpha)$	α is a threshold: if $\alpha>\alpha_{jl}$, the association is nonbinding, if $\alpha<\alpha_{jl}$ the association is binding	[10]
$J_5=\sum_{i=1}^{n}\frac{1}{N_i(N_i-1)}\sum_{\substack{j,l\in B_i\\ j\neq l}}\alpha_{jl}$	If N = 1, then $\frac{1}{N_i-1}\sum_{\substack{j,l\in B_i\\ j\neq l}}\alpha_{jl}=0$	[1]
$J_6=\frac{1}{m}\sum_{i=1}^{n}\frac{1}{N_i-1}\sum_{\substack{j,l\in B_i\\ j\neq l}}\alpha_{jl}$	m is the number of elements in each aggregated group	[1]

assigned to a division which satisfy the divisional technology constraints; 3) the total cost of implementing the solution should not exceed the allocated resources.

Some examples of headquarters objective functionals are given in Table 1.

3. A Mathematical Model and Solution of the Problem

We start by introducing some notation. $H(x)$ is the headquarters objective function; $h_i(\mathbf{y}_i)$ is the payoff function of the i-th division; Y_i is the set of alternative MSDs which may be assigned to division i given the technology constraints; $\mathbf{x}_i=(x_{ij})$, where $\mathbf{x}_i$ is the plan vector assigned by the headquarters to division i; $\mathbf{G}_i(\mathbf{x}_i)$ is the vector function of the technology constraints; $\mathbf{b}_i$ is a column vector; $x_{ij}=1$, if the j-th MSD is assigned by the headquarters to division i, $x_{ij}=0$ otherwise; $\mathbf{y}_i=(y_{ij})$, where $y_{ij}=1$ if the j-th MSD is preferred by the division i, $y_{ij}=0$ otherwise; $\chi_i(\mathbf{x}_i,\mathbf{y}_i)$ is the function of side payments independent of the plan, $\chi_i(\mathbf{x}_i,\mathbf{y}_i)=\lambda_i$ if $i\neq i$, $\chi_i(\mathbf{x}_i,\mathbf{y}_i)=0$ otherwise; λ_i are the side payments earned by compliance with the headquarters decisions, and their sum is bounded by some given number λ_0; S is the set of coordinated MSDs,

Here S_i is the set of coordinated MSDs for division i,

$$S_i=\{\mathbf{x}_i\,|\,h_i(\mathbf{x}_i)\geqslant\max_{\mathbf{y}_i\in Y_i}[h_i(\mathbf{y}_i)-\chi_i(\mathbf{x}_i,\mathbf{y}_i)]\}. \tag{1}$$

The model of the design problem for a coordinated MS now can be written in the following form:

$$H(\mathbf{x})\to\min, \tag{2}$$

$$\mathbf{G}_i(\mathbf{x}_i)=\mathbf{b}_i,\quad i=\overline{1,n}, \tag{3}$$

$$\sum_{i=1}^{n} x_{ij}=1, \quad j=\overline{1, m}, \tag{4}$$

$$\sum_{j=1}^{m} x_{ij}\geqslant 1, \quad i=\overline{1, n}, \tag{5}$$

$$x_{ij}\in\{0, 1\}, \quad i=\overline{1, n}, \quad j=\overline{1, m}, \tag{6}$$

$$h_i(\mathbf{x}_i)\geqslant \max_{\mathbf{y}_i\in Y_i} h_i(\mathbf{y}_i)-\lambda_i, \quad i=\overline{1, n}, \tag{7}$$

$$\sum_{i=1}^{n}\lambda_i\leqslant\lambda_0. \tag{8}$$

The constraints (7) are obtained from (1) using the structure of $\chi_i(x_i, y_i)$ [4]; Eq. (4) implies that each MSD is assigned to only one division; relationship (3) is a divisional technology constraint; the inequality (5) indicates that at least one MSD is assigned to each division.

Problem (2)-(8) is solved by an algorithm which uses subalgorithms exploiting the specific features of each concrete problem. Note tht the accuracy of the algorithm depends on the particular subalgorithm employed.

Let X be the set defined by the constraints (3)-(6). We will describe the k-th step of the algorithm, initially setting $k = 1$, $R_k = \phi$.

1°. Find $h_i^* = \max h_i(y_i)$ over $y_i \in Y_i$, $i = 1, n$.

2°. Find $\mathbf{x}^* = \operatorname{argmin} H(\mathbf{x})$ over $\mathbf{x} \in X \quad R_k$ by one of the standard methods, e.g., the method of [5]. $R_{k+1} = R_k \cup \{x^*\}$.

3°. Find λ_i by the formula

$$\lambda_i=h_i^*-h_i(\mathbf{x}_i^*), \quad i=\overline{1, n}. \tag{9}$$

4°. Check the constraint (8); if it is satisfied, then end; else $k = k + 1$ and go to 2°.

Let us consider the problem (2)-(8) with

$$H(\mathbf{x})\rightarrow\max \tag{10}$$

when the function $H'(\mathbf{x}^*)=\sum_{i=1}^{n} h_i(\mathbf{x}_i^*)$ where $\mathbf{x}^*=(\mathbf{x}_i^*)$ is a solution of problem (10), (3)-(6). Clearly $H(\mathbf{x})$ and $H'(\mathbf{x})$ are defined on the same partially ordered set X.

For the problem (10), (3)-(8) we have the following:

<u>Proposition</u>. Let $H'(\mathbf{x})$ be a monotone function of $H(\mathbf{x})$ [6]. If the problem (10), (3)-(8) has a solution, then the problem is epicoordinated.

The proof is given in the appendix.

Examples of problems and functions which have the monotonicity property are given in the next section.

Note that:

1) simultaneously with a coordinated plan of the problem (10), (3)-(8), we determine the minimum total side payments;

2) the proposition remains valid when the problem (10), (3)-(8) involves simultaneous minimization of the headquarters and divisional objective functions;

3) the proposition remains valid when the division solves the knapsack problem, which is the most general integer programming problem [7];

4) the results are also applicable to the problem (2)-(4), (6)-(8). Assume that a solution of this problem exists and that for some divisions $\sum_{j=1}^{m} x_{ij}=0$, $i \in I_1$, $|I_1| < n$. This corresponds to the case when the number of MBs in the MS should be reduced;

5) the problem

$$H(\mathbf{x}) \to \max, \tag{11}$$

$$g_i(\mathbf{x}_i) \leqslant \mathbf{b}_i, \quad i=\overline{1, n}, \tag{12}$$

$$h_i(\mathbf{x}_i) \geqslant \max_{\mathbf{y}_i \in Y_i} (h_i(\mathbf{y}_i) - \chi_i(\mathbf{x}_i, \mathbf{y}_i)), \quad i=\overline{1, n}, \tag{13}$$

where Y_i is defined by the constraint (12), $g_i(\mathbf{x}_i)$ is the technology constraint function of the division i, and $\mathbf{x}$ are possibly continuous, is epicoordinated in the following cases:

a) $H'(\mathbf{x})$ is a monotone function of $H(\mathbf{x})$, $\chi_i(\mathbf{x}_i, \mathbf{y}_i)$ are penalties of type NP [4];

b) $h_i(\mathbf{x}_i)$ are monotone functions of $g_i(\mathbf{x}_i)$, the constraint (12) is active for the optimal solution of the problem (11)-(12), and $\chi_i(\mathbf{x}_i, \mathbf{y}_i)$ are arbitrary penalty functions.

4. Relationship with Aggregation Problems

The design problem for MS and for organizational structures is often solved using models and methods of some aggregation problems. The design problem may be restated in terms of the aggregation problem and solved accordingly. A specific feature of the design problem is that each pair of MSDs (j, ℓ) is characterized by a number $\alpha_{j\ell}$, which is interpreted as the "association strength" or a measure of "closeness" of the two MSDs, i.e., the given set of MSDs is characterized by the matrix $\mathbf{A} = (\alpha_{j\ell})_{j,\ell=1}{}^{m}$, which we call the association matrix.

The mathematical model of the problem restated as a 0–1 programming problem coincides with the mathematical model (2)-(8). The problem is solved by an algorithm which differs from that of Sec. 3 in that $\mathbf{x}^*$ in step 2° is determined by one of the methods used for aggregation problems.

While the division determines the h_i^* on its own association matrix, the headquarters solves the MS design problem by first partitioning the MSD set into MBs and then choosing the best assignment of MBs to each division. The assignment problem of MBs to division subject to additional constraints has the following form:

$$H(\mathbf{z}) = \sum_{i=1}^{n} \sum_{r=1}^{n} \lambda_{ir} z_{ir} \to \min, \tag{14}$$

$$\sum_{i=1}^{n} z_{ir} = 1, \quad r=\overline{1, n}, \tag{15}$$

$$\sum_{r=1}^{n} z_{ir} = 1, \quad i=\overline{1, n}, \tag{16}$$

$$\mathbf{G}_i'(\mathbf{z}_i) \leqslant \mathbf{b}_i, \quad i=\overline{1, n}, \tag{17}$$

$$z_{ir} \in \{0, 1\}, \quad r=\overline{1, n}, \quad i=\overline{1, n}, \tag{18}$$

where $\lambda_i = (\lambda_{ir})$, λ_{ir} are the side payments to the division i associated with the assignment of the MB r to this division; $\mathbf{z} = (\mathbf{z}_i)$, $\mathbf{z}_i = (z_{ir})$, where $z_{ir} = 1$ if the r-th MB is assigned to division i, $z_{ir} = 0$ otherwise; $\mathbf{G}_i'(\mathbf{z}_i)$ is the vector function of technology constraints.

We describe the k-th step of the algorithm, initially setting $k = 1$, $R_k = \phi$.

1°. Find $h_i^* = \max h_i(\mathbf{y}_i)$ over $\mathbf{y}_i \in Y_i$, $i=\overline{1, n}$.

2°. Using one of the methods of solution of aggregation problems, find $\mathbf{x}^* = \operatorname{argmin} H(\mathbf{x})$ over $\mathbf{x} \in X \setminus R_k$, $R_{k+1} = R_k \cup \{\mathbf{x}^*\}$. Here i is the MB index.

3°. Solve the problem (14)-(18). Check the constraint. If it is satisfied then end; else $k = k + 1$ and go to 2°.

Aggregation problems are mostly solved by heuristic methods. Therefore, the accuracy of the proposed algorithm may be improved only by increasing the number of enumerated alternatives. Directional enumeration is ensured by selecting branching vertices which correspond to the best values of the objective function. The number of branching vertices for $n = 2$ is given by the formula $2^{m-1} - 1$. In order to speed up the algorithm, we may choose in step 3° only those solutions which satisfy the constraint (17).

Table 1 presents some functionals used in the aggregation problem. Let us examine them more closely. If the functionals J_1, J_3, J_4, J_5, J_6 are defined on the same partially ordered set X, then they are monotone functions of each other, since any of them may be obtained from any other functional by monotone transformations [6]. Consider the following examples: a) The functional J_4 is transformed to J_3 by subtracting a number α (the threshold) from each element of the association matrix and, conversely, J_3 is obtained from J_4 by adding the number α to each element of the association matrix; b) decompose the functional J_3 into terms; each of the terms is a monotone function. If we now form a linear combination of the terms with the coefficients k_i, then we again obtain a monotone function. These transformations reduce the functional J_3 to: a) J_5 when $k_i = 1/N_i(N_i - 1)$; b) J_1 when $k_i = 1/N_i$, etc. Other transformations can be performed similarly. Note that the monotonicity property is symmetrical for the functionals of Table 1.

Thus, using one of the functionals from Table I as the headquarters objective function and the components of some other functional as the divisional objective functions, we conclude that if the problem (10), (3)-(8) has a solution, then it is epicoordinated.

The functional J_2 is not a monotone function of the other functionals in Table 1. When it is paired with any of the functionals J_1, J_3-J_6, we have to solve the problems (2)-(8) or (10), (3)-(8) by the above algorithm.

Note that the results may be applied to design a coordinated MS with specialized MBs and MSDs organized for group processing of parts and group manufacturing processes [2].

The results of this paper will be applied to design the MS of the subdivisions in one of the instrument-building plants.

5. Conclusion

We applied the principle of coordinated planning to solve the MS design problem. The results may be generalized to the design of MS in multilevel systems. Another possible application of our results is to the design of coordinated organizational structures [8]. The problem can be solved by methods developed for the more general coordinated planning problem with 0–1 variables, which is the subject of future research and requires special analysis.

APPENDIX

1. Proof of Proposition. Let $\mathbf{x}^*$ be an optimal solution of the problem (10), (3)-(6) and let λ^* be given by (9). Then, in order to prove the proposition, it suffices to show that if the constraint (8) does not hold for $\mathbf{x}^*$ and λ^*, then the problem (10), (3)-(8) has no solution.

Summing (7) over all i, we obtain

$$\sum_{i=1}^{n} \max_{\mathbf{y}_i \in Y_i} h_i(\mathbf{y}_i) - \sum_{i=1}^{n} \lambda_i \leq \sum_{i=1}^{n} h_i(\mathbf{x}_i). \tag{A.1}$$

Since $H'(\mathbf{x}) = \sum_{i=1}^{n} h_i(\mathbf{x}_i)$ is a monotone function of $H(\mathbf{x})$, passing from $\mathbf{x}^*$ to any assignment $\mathbf{x}'$, we have $\sum_{i=1}^{n} h_i(\mathbf{x}_i^*) \geqslant \sum_{i=1}^{n} h_i(\mathbf{x}_i')$ since $H(\mathbf{x}^*) \geq H(\mathbf{x}')$. Now, since the first term in the left-hand side of (A.1) is constant, the constraint (8) a fortiori does not hold for $\mathbf{x}'$. Q.E.D.

2. <u>Solving the Design Problem for a Coordinated MS</u>. Consider the following example:

$$H(\mathbf{x})=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{\substack{l=1\\l>j}}^{m}\alpha_{lj}x_{ij}x_{il}\to\max, \tag{A.2}$$

$$1\leqslant\sum_{j=1}^{m}x_{ij}\leqslant \mathrm{b}_i,\qquad i=1,n, \tag{A.3}$$

$$\sum_{i=1}^{n}x_{ij}=1,\qquad j=\overline{1,m}, \tag{A.4}$$

$$x_{ij}\in\{0,1\},\qquad i=\overline{1,n},\qquad j=\overline{1,m}, \tag{A.5}$$

$$\sum_{j=1}^{m}\sum_{l=1}^{m}r_{lj}{}^{i}x_{ij}x_{il}=\max_{y_i\in Y_i}\sum_{j=1}^{m}\sum_{\substack{l=1\\l>j}}^{m}r_{lj}{}^{i}y_{ij}y_{il}-\lambda_i,\qquad i=\overline{1,n}, \tag{A.6}$$

$$y_{ij}\in\{0,1\},\qquad i=\overline{1,n},\qquad j=\overline{1,m}, \tag{A.7}$$

$$\sum_{i=1}^{n}\lambda_i\leqslant\lambda_0, \tag{A.8}$$

where $Y_i=\left\{\mathbf{y}_i/1\leqslant\sum_{j=1}^{m}y_{ij}\leqslant\mathbf{b}_i\right\}$, $b_i=3$, $i=\overline{1,n}$, $n=2$, $m=5$, $\lambda_0=40$. The MSD association matrix is taken in the form

$$\mathbf{A}=(\alpha_{lj})=\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5\\ \hline 1 & 0 & 20 & 15 & 8 & 15\\ 2 & 0 & 0 & 31 & 15 & 10\\ 3 & 0 & 0 & 0 & 14 & 3\\ 4 & 0 & 0 & 0 & 0 & 8\\ 5 & 0 & 0 & 0 & 0 & 0\end{array},\quad R_1=(r_{lj}^1)=\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5\\ \hline 1 & 0 & 15 & 20 & 11 & 13\\ 2 & 0 & 0 & 8 & 4 & 20\\ 3 & 0 & 0 & 0 & 13 & 8\\ 4 & 0 & 0 & 0 & 0 & 15\\ 5 & 0 & 0 & 0 & 0 & 0\end{array},$$

$$R_2=(r_{lj}^2)=\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5\\ \hline 1 & 0 & 20 & 11 & 15 & 16\\ 2 & 0 & 0 & 8 & 13 & 25\\ 3 & 0 & 0 & 0 & 4 & 5\\ 4 & 0 & 0 & 0 & 0 & 13\\ 5 & 0 & 0 & 0 & 0 & 0\end{array}.$$

Initially set $k = 1$, $R_1 = \phi$.

<u>1st Iteration</u>.

1°. Solve the problem $h_i^*=\max\sum_{j=1}^{m}\sum_{\substack{l=1\\l>j}}^{m}r_{lj}{}^{i}y_{ij}y_{il}$ over $y_i \in Y_i$. We have $h_1^* = 48$, $y_{11} = y_{12} = y_{15} = 1$, $h_2^* = 61$, $y_{21} = y_{22} = y_{25} = 1$.

2°. Solve the problem (A.2)-(A.5) with $H(\mathrm{x})\to\max$ over $x\in X\setminus R_1$, using the "pooling" method [1]. The intermediate results are represented by the matrices A_{11} and A_{12}, where A_{11} denotes the association matrix obtained in iteration 1 after step 1. In order to generate A_{11}, we chose α_{23}. In order to generate A_{12}, we chose $\alpha_{1,(2,3)}$. The rows and the columns in these matrices may have multiple indices, which represents pooling of rows and columns corresponding to the matrix element selected in the preceding iteration:

$$A_{11}=\begin{array}{c|cccc} & 1 & 2,3 & 4 & 5\\ \hline 1 & 0 & 17{,}5 & 8 & 15\\ 2,3 & 0 & 0 & 14{,}5 & 6{,}5\\ 4 & 0 & 0 & 0 & 8\\ 5 & 0 & 0 & 0 & 0\end{array},$$

$$A_{12}=\begin{array}{c|ccc} & 1,2,3 & 4 & 5\\ \hline 1,2,3 & 0 & 12{,}3 & 8{,}3\\ 4 & 0 & 0 & 8\\ 5 & 0 & 0 & 0\end{array}.$$

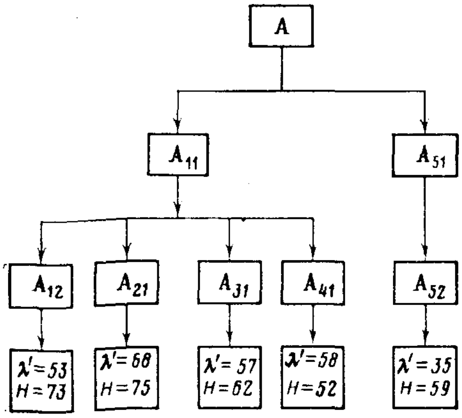

Fig. 1. Schematic diagram illustrating the solution of the example.

We finally obtain the following composition of the manufacturing blocks: MB1 = {1, 2, 3}, MB2 = {4, 5}, $x_{11}^* = x_{12}^* = x_{13}^* = x_{24}^* = x_{25}^* = 1$, H = 74, $R_2 = R_1 \cup \{x^*\}$. The remaining $x_{ij}^* = 0$. In what follows, x_{ij}^* will be omitted. The process of branching from A to A_{11} and from A_{11} to A_{12} is shown in Fig. 1, where $\lambda' = \sum_{i=1}^{n} \lambda_i$.

3°. Solve the MB assignment problem. As a result, the first division is assigned MB1, the second division MB2, with $\lambda_1 + \lambda_2 = 53$. Since $53 > \lambda_0$, go to step 2° setting k = 2.

2nd Iteration.

2°. Solve the problem (A.2)-(A.5) with $H(\mathbf{x}) \to \max$ over $\mathbf{x} \in X \setminus R_2$. In this case, it suffices to start the branching with A_{11} (Fig. 1). Choose α_{15} and find A_{21}. This gives

$$A_{21} = \begin{array}{c} \\ 1,5 \\ 2,3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1,5 & 2,3 & 4 \end{array} \\ \begin{bmatrix} 0 & 12 & 8 \\ 0 & 0 & 14,5 \\ 0 & 0 & 0 \end{bmatrix} \end{array}, \quad A_{31} = \begin{array}{c} \\ 1,4 \\ 2,3 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1,4 & 2,3 & 0 \end{array} \\ \begin{bmatrix} 0 & 16 & 11,5 \\ 0 & 0 & 6,5 \\ 0 & 0 & 0 \end{bmatrix} \end{array}.$$

3°. Assign MB2 to the first division, MB1 to the second division. Find $\lambda_1 + \lambda_2 = 68 > \lambda_0$, set k = 3, and go to step 2°.

3rd Iteration

2°. Solve the problem (A.2)-(A.5) with $H(\mathbf{x}) \to \max$ over $\mathbf{x} \in X \setminus R_3$. Choose $\alpha_{1,4}$ in A_{11}. Generate the vertex corresponding to the matrix A_{31} (Fig. 1). Finally, MB1 = {1, 4, 5}, MB2 = {2, 3}, H = 62, $R_4 = R_3 \cup \{x^*\}$.

3°. Assign MB2 to the first division, MB1 to the second division. Find $\lambda_1 + \lambda_2 = 57 > \lambda_0$, set k = 4, and go to step 2°.

4th Iteration

2°. Solve the problem (A.2)-(A.5) with $H(\mathbf{x}] \to \max$ over $\mathbf{x} \in X \setminus R_4$. Choose $\alpha_{(2,3),5}$ in A_{11}. Generate the vertex corresponding to the matrix A_{41} (Fig. 1). Finally, MB1 = {2, 3, 5}, MB2 = {1, 4}, H = 52, $R_5 = R_4 \cup \{x^*\}$. The matrix A_{41} is

$$A_{41} = \begin{array}{c} \\ 1 \\ 2,3,5 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1 & 2,3,5 & 4 \end{array} \\ \begin{bmatrix} 0 & 21,5 & 8 \\ 0 & 0 & 18,5 \\ 0 & 0 & 0 \end{bmatrix} \end{array}.$$

3°. Assign MB1 to the first division, MB2 to the second division. Find $\lambda_1 + \lambda_2 = 58 > \lambda_0$. Set k = 5, and go to step 2°.

5th Iteration

2°. Solve the problem (A.2)-(A.5) with $H(\mathbf{x}) \to \max$ over $\mathbf{x} \in X \setminus R_5$. Choose α_{12} in A, generate the vertex corresponding to the matrix A_{51}. In A_{51} choose α_{34} and generate the vertex corresponding to the matrix A_{52}. Finally, MB1 = {1, 2, 5}, MB2 = {3, 4}, H = 59, $R_6 = R_5 \cup \{x^*\}$. The matrices are

$$A_{51} = \begin{array}{c} \\ 1,2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{cccc} 1,2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 23 & 11,5 & 12,5 \\ 0 & 0 & 14 & 3 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}, \quad A_{52} = \begin{array}{c} \\ 1,2 \\ 3,4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccc} 1,2 & 3,4 & 5 \end{array} \\ \begin{bmatrix} 0 & 17,2 & 12,5 \\ 0 & 0 & 5,5 \\ 0 & 0 & 0 \end{bmatrix} \end{array}.$$

3°. Assign MB2 to the first division and MB1 to the second division. We have $\lambda_1 + \lambda_2 = 35 < \lambda_0$, so that the result is a coordinated MS.

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