COORDINATED PLANNING IN A DISCRETE ACTIVE SYSTEM

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The problem of coordinated planning of the production divisions in a discrete active system is considered and the corresponding mathematical model is constructed. The properities of the set of coordinated solutions are identified and studied. A solution method is proposed and its complexity is estimated. The paper reports the results of one of the first attempts to apply the principles of coordinated planning in a small-batch multiproduct enterprise (development and installation of a computerized on-line/long-term coordinated planning system for a stamping shop in an instrument-building plant).

1. INTRODUCTION

Coordinated optimization of two-level hierarchical systems (HS) is a popular area of current research [1, 2]. Solution methods of coordinated planning (CP) problems allowing for the goal-directed behavior of the lower-level subsystems in the HS are among the topics considered in this area.

The HS functioning mechanisms are studied in two directions in the existing literature [1-13]. One direction focuses on analysis and synthesis of functioning mechanisms of two-level HS [1-4]. The corresponding work is surveyed in [5], where the following main results are noted: 1) construction of functioning mechanisms ensuring prescribed coordinated functioning regimes of active systems (AS); 2) determination of necessary and sufficient conditions of optimality of coordinated functioning regimes subject to given control performance criteria [1, 4, 5].

The second direction focuses on models and numerical methods for the solution of some discrete coordinated optimization problems of two-level HS for given functioning mechanisms under conditions of complete information at the center [6-13]. First CP models and optimization methods were proposed in [6]. Exact algorithms for the solution of combinatorial CP problems (coordinated scheduling problems) in a two-level AS with dependent active elements (AE) were published in [4, 7-9]. An approximate algorithm for the allocation of discrete resources in a HS was described in [10]. Solution algorithms for discrete coordination problems in HS combining the methods of vector optimization and sequential analysis of alternatives were proposed in [11]. Synthesis of a coordinated AS structure, which is a particular case of the problems considered in this paper, was studied in [12, 13].

The present study (which can be assigned to the second direction of research) develops a substantive formulation, a model, and a numerical solution method for the discrete CP problem in a tow-level AS with fanlike structure.

2. STATEMENT OF THE PROBLEM AND THE MATHEMATICAL MODEL

Consider an industrial enterprise with discrete production. We represent it as a two-level AS, consisting of the management (the center) and the subordinated production divisions (the active elements). The production divisions function as follows. The main task of the production teams is to manufacture batches of parts according to a monthly plan, using blanks stocked in the divisional store. Production deadlines must be strictly observed, because this determines the continuity of operation of the assembly teams in the enterprise. In other words, the batches with the earliest deadlines should be included by the production teams in their daily-shift assignments. This primarily serves the interests of the whole enterprise, so that this problem can be viewed as the problem of the center. With

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tight monthly schedules and limited production capacity, the production teams naturally tend to give priority to batches that have the highest cost. This is attributable to the specific incentives system instituted in the enterprise, which evaluates the activity of production teams only in terms of the fulfillment of the necessary product mix and normed assignment. The production teams thus develop the daily-shift assignments so as to maximize their objective functions, which are determined by the material incentive system. The parameters of the incentive system are known to the production teams. We call this problem the AE problem. Then the production teams can be identified with the AEs, and the production team of each shop or division has its own goals and preferences.

The existence of different preferences for the center and the AEs requires coordination. Moreover, both the center and the AEs attempt to utilize as fully as possible the divisional production capacity, which requires minimizing the auxiliary jobs* needed for the completion of each batch.

The AE operations planning accordingly starts with coordinating (by volume and product mix) the daily-shift assignments. Then the assignment is broken down into a more detailed plan, i.e., a processing schedule is constructed for the batches included in the coordinated daily-shift assignment so as to minimize the auxiliary jobs for the divisional machines. The planning problem for the production divisions thus can be divided into two subproblems.

1. The problem of determining the coordinated daily-shift assignments (a CP problem), which is formulated as follows: Include in the daily-shift assignments the batches with the earliest deadlines, subject to AE preferences and technological constraints.

2. The problem of optimal scheduling of the machines required for processing the parts included in the coordinated daily-shift assignment. The substantive formulation and the solution methods for this problem are described in detail in [14], and in what follows we can consider only Problem 1.

We introduce the following notation: 1) $\mathbf{x}=(\mathbf{x}_1,\ldots,\mathbf{x}_i,\ldots,\mathbf{x}_n)$ is the daily-shift assignment for the AS (a n_0-dimensional composite 0-1 vector), where $\mathbf{x}_i=(x_{i1},\ldots,x_{ij},\ldots,x_{im_i})$ is the daily-shift assignment of the i-th AE (a m_i-dimensional 0-1 vector); $x_{ij}=1$ if the j-th batch is included in the assignment of the i-th AE, $x_{ij}=0$ otherwise; 2) $\mathbf{y}_i$ is the actual performance of the daily-shift assignment by the i-th AE (a 0-1 vector), $\mathbf{y}_i=(y_{i1},\ldots,y_{ij},\ldots,y_{im_i})$, where $y_{ij}=1$ if the j-th batch is actually included in the daily-shift assignment of the i-th AE and $y_{ij}=0$ otherwise; 3) $X\subset R^{n_0}$ is the set of feasible shift assignments in the AS, determined by the production constraints; $H_i\in R^{m_i}$ is the actual performance set of the shift assignments of the i-th AE determined by technological constraints; 4) $\Phi(x)=\sum_{i=1}^{n}\mathbf{a}_i^T\mathbf{x}_i$ is a linear objective function expressing the loss to the AS due to failure to meet the deadlines; $\mathbf{a}_i$ is the vector of coefficients of the AS loss function attributable to the violation of deadlines by the i-th AE; $f_i(\mathbf{x}_i,\mathbf{y}_i)=h_i(\mathbf{y}_i)-\chi_i(\mathbf{x}_i,\mathbf{y}_i)$ is the incentive system of the i-th AE; $h_i(y_i)=\mathbf{c}_i^T\mathbf{y}_i$ is the payoff function of the i-th AE, where $\mathbf{c}_i$ is the vector coefficients of the payoff function of the i-th AE, $\chi_i(\mathbf{x}_i,\mathbf{y}_i)=(\chi_1(\cdot),\ldots,\chi_j(\cdot)\ldots,\chi_{m_i}(\cdot))$ is the penalty vector function of the i-th AE, T denotes the transpose; 5) $S_i=\{\mathbf{x}_i|h_i(\mathbf{x}_i)\geqslant\max[h_i(\mathbf{y}_i)-\chi_i(\mathbf{x}_i,\mathbf{y}_i)]$ over $\mathbf{y}_i\in H_i\}$ is the set of coordinated plans of the i-th AE.

Then the mathematical model of the CP problem has the form

$$\Phi(\mathbf{x})=\sum_{i=1}^{n}\mathbf{a}_i^T\mathbf{x}_i\to\min_{\mathbf{x}\in X\cap S},$$

$$S=\prod_{i=1}^{n}S_i,$$

$$S_i=\{\mathbf{x}_i|\mathbf{c}_i^T\mathbf{x}_i\geqslant\max_{\mathbf{y}_i\in H_i}(\mathbf{c}_i^T\mathbf{y}_i-\mathbf{e}_i^T\chi_i(\mathbf{x}_i,\mathbf{y}_i))]\},\quad i=\overline{1,n},\tag{1}$$

$$X=\{\mathbf{x}|\mathbf{x}=(\mathbf{x}_1,\ldots,\mathbf{x}_i,\ldots,\mathbf{x}_n),\quad \mathbf{x}_i=(x_{i1},\ldots,x_{ij},\ldots,x_{im_i}),$$

*Auxiliary jobs in this context include setup and adjustment of machines and transportation of blanks and semifinished parts from one machine to another.

$$Ax \geqslant p,\ x_{ij} \in \{0, 1\}\}.$$

$$H_i = \{y_i \mid y_i = (y_{i1}, \dots, y_{ij}, \dots, y_{im_i}),\quad B_i y_i \leqslant b_i,$$

$$y_{ij} \in \{0, 1\}\},\quad i = \overline{1, n},$$

where e_i is the m_i-dimensional unit vector, b_i is the L_i-dimensional column vector determining the production capabilities of the i-th AE, B_i is the $L_i \times m_i$ matrix of technological coefficients of the i-th AE. The components of the matrices A, B_i (i = 1, ..., n) and the vectors p, c_i^T, b_i, a_i^T $(i = \overline{1, n})$ are assumed nonzero positive.

In what follows, the problem $\min \Phi(x)$ over $x \in X$ will be called the Φ-problem, the problem $\max[h_i(y_i) - \chi_i(x_i, y_i)]$ over $y_i \in H_i$ will be called the i-th AE problem, and the problem $\max h_i(y_i)$ over $y_i \in H_i$ will be called the P_i-problem.

The solution of the CP problem (1) is difficult because of the nonstandard constraints describing the sets S_i (i = 1, ..., n). One of the known approaches to the solution of such problems involves preliminary construction of the set S_i (i = 1, ..., n) followed by solving standard mathematical programming problems.

3. CONSTRUCTING THE SET OF COORDINATED PLANS

Using the results of Theorem 3 [15] and noting that the set S is the Cartesian product of the sets S_i corresponding to the coordinated plans of the different AEs, we can replace the n independent AEs with one generalized AE and reduce the CP problem (1) to a problem with one generalized AE. In what follows, unless otherwise specified, we omit the index i of the set and the variables.

Construction of the set S requires specializing the penalty function. Consider a penalty function which is independent of the plan [1]:

$$\chi(x, y) = \sum_{j=1}^{m} \chi_j(x_j, y_j),\quad \chi_j(x_j, y_j) = \begin{cases} \alpha_j, & \text{if } y_j > x_j, \\ 0, & \text{if } y_j = x_j, \\ \beta_j, & \text{if } y_j < x_j, \end{cases} \tag{2}$$

where α_j and β_j are the penalty function coefficients (α_j, β_j = const).

Note that if x and y are 0-1 vectors, then the penalty function (2) is equivalent to the linear penalty function

$$\chi(x, y) = \sum_{j=1}^{m} \chi_j(x_j, y_j) = \sum_{j \in J_1} \alpha_j(y_j - x_j) + \sum_{j \in J_2} \beta_j(x_j - y_j),$$

where $J_1 = \{j \mid x_j = 0\}$ and $J_2 = \{j \mid x_j = 1\}$. In what follows, we consider linear penalty functions. In our case, without loss of generality, we may set $\alpha_j = \beta_j$, j = 1, ..., m. The set S is constructed for the case when

$$\alpha_j = \varepsilon c_j,\ 0 < \varepsilon < 1,\ j = \overline{1, m}. \tag{3}$$

We use the following definitions.

The weight $p(x)$ of the 0-1 vector $x = (x_1, \dots, x_m)$ is defined as

$$p(x) = \sum_{j=1}^{m} x_j.$$

The distance between the vectors x and y is defined by the Hamming metric $d(x, y) = \sum_{j=1}^{m} |x_j - y_j|$.

The set Q_x^{l-1} is the set of vectors y for which $p(y) = l-1$, $d(x, y) = 1$, where $l = p(x)$, i.e., $Q_x^{l-1} = \{y \mid p(y) = l-1,\ d(x, y) = 1\}$.

The set Q_x^{l+1} is the set of vectors y for which $p(y) = l+1$, $d(x, y) = 1$, where $l = p(x)$, i.e., $Q_x^{l+1} = \{y \mid p(y) = l+1,\ d(x, y) = 1\}$.

The upper-level Boolean set F_0 is the set of vectors $\mathbf{y}\in H$ such that $Q_{\mathbf{y}}^{l+1}\cap H=\phi$, i.e., $F_0=\{\mathbf{y}\in H \mid Q_{\mathbf{y}}^{l+1}\cap H=\phi\}$.

LEMMA 1. If condition (3) is satisfied for the CP problem (1), then the upper-level set F_0 includes the set of coordinated plans S, i.e., $S \subseteq F_0$.

Lemma 1 and all the subsequent theorems are proved in the Appendix.

It follows from Lemma 1 that if $\mathbf{x}\in F_0$ and $\mathbf{x}\notin S$ (x is the prohibited vector), then no vector $\mathbf{y}\in S$ exists such that $d(\mathbf{x},\mathbf{y})=1$.

Change the indices j so that if $j_1 < j_2$, then $c_{j1} \leq c_{j2}$. We say that the vector $\mathbf{x}^0$ is in relation $\overset{\omega}{\rightarrow}$ with the vector $\mathbf{y}$ if $x_{j_1}^0=1$, $x_{j_2}^0=0$, $y_{j_1}=0$, $y_{j_2}=1$, $\forall j\neq j_1, j_2$, $x_j^0=y_j$ and $c_{j1} < c_{j2}$. For the vector $\mathbf{x}^0$ we define the following sets:

1) $V_{\mathbf{x}^0}^l=\{\mathbf{y} \mid p(\mathbf{y})=l,\ d(\mathbf{x}^0,\mathbf{y})=2\}$,

2) $V_{\mathbf{x}^0}^{lp}=\{\mathbf{y}\in V_{\mathbf{x}^0}^l \mid \mathbf{x}^0 \overset{\omega}{\longrightarrow} \mathbf{y}\}$,

3) $V_{\mathbf{x}^0}^{ld}=\{\mathbf{y}\in V_{\mathbf{x}^0}^l \mid \mathbf{y} \overset{\omega}{\longrightarrow} \mathbf{x}^0\}$.

Note that $V_{\mathbf{x}^0}^l=V_{\mathbf{x}^0}^{lp}\cup V_{\mathbf{x}^0}^{ld}$ and $V_{\mathbf{x}^0}^{lp}\cap V_{\mathbf{x}^0}^{ld}=\phi$.

THEOREM 1. Let $\mathbf{x}^0\in S$. if $\mathbf{x}\in V_{\mathbf{x}^0}^{lp}\cap F_0$, then $\mathbf{x}\in S$.

Consider the graph $G_\ell = (W_\ell, U)$, where

$$W_l=\{\mathbf{x} \mid p(\mathbf{x})=l\},\ U=\{u_{\mathbf{x},\mathbf{y}} \mid d(\mathbf{x},\mathbf{y})=2\}.$$

Take the subgraph $G_\ell^S = (W_\ell^S, U^S)$, where

$$W_l^s=\{\mathbf{y}\in W_l \mid \mathbf{y}\in S\},\ U^s=\{u_{\mathbf{x},\mathbf{y}}\in U \mid \mathbf{x}\in S,\ \mathbf{y}\in S,\ d(\mathbf{x},\mathbf{y})=2\}.$$

<u>COROLLARY 1</u>. Let $W_\ell^S \neq \emptyset$. Then the subgraph G_ℓ^S is connected.

<u>COROLLARY 2</u>. Let $\mathbf{x}^*\in V_{\mathbf{x}^0}^{ld}\cap F_0$, where $\mathbf{x}^0\in S$ and $\mathbf{x}^*\notin S$. if $\mathbf{x}\in V_{\mathbf{x}^*}^{ld}\cap F_0$, then $\mathbf{x}\notin S$.

Theorem 1 directly implies that the set of vectors obtained from the vector $\mathbf{x}^0\in S$ by successive application of the relation $\overset{\omega}{\rightarrow}$ is coordinated.

The set S is constructed as follows. Assume that we have found the vector $\mathbf{x}^0=(x_1^0,\ldots,x_m^0)\in S$. Determine the set $V_{\mathbf{x}^0}^l=V_{\mathbf{x}^0}^{lp}\cup V_{\mathbf{x}^0}^{ld}$. By Theorem 1, $V_{\mathbf{x}^0}^{\ell p} \subseteq S$. Index the vectors of the set $V_{\mathbf{x}^0}^{ld}$ by μ ($\mu = 1, .., N$) so that if $\mu_1 < \mu_2$, then $h(\mathbf{x}^{\mu_1})\leqslant h(\mathbf{x}^{\mu_2})$. Perform one of the following operations for each vector $\mathbf{x}^\mu$ ($\mu = 1, \ldots, N$) in the order of inceasing indices:

a) if $\mathbf{x}^\mu\notin H^2$ determine the set $Q_{\mathbf{x}^\mu}^{l-1}$, where $Q_{\mathbf{x}^\mu}^{l-1}=\{\mathbf{x} \mid p(\mathbf{x})=l-1,\ d(\mathbf{x}^\mu,\mathbf{x})=1\}$,

b) if $\mathbf{x}^\mu\in H, \mathbf{x}^\mu\notin F_0$ determine the set $Q_{\mathbf{x}^\mu}^{l+1}$, where $Q_{\mathbf{x}^\mu}^{l+1}=\{\mathbf{x} \mid p(\mathbf{x})=l+1,\ d(\mathbf{x}^\mu,\mathbf{x})=1\}$,

c) if $\mathbf{x}^\mu\in S$ then by connectivity of the graph G_ℓ^S determine the set $V_{\mathbf{x}^\mu}^l$,

d) if $\mathbf{x}^\mu\in F_0, \mathbf{x}^\mu\notin S$, then the vector $\mathbf{x}^\mu$ is prohibited.

The procedure of construction of the set S continues while at least one allowed vector $\mathbf{x}^\mu\notin S$ remains.

The construction of the set S requires testing for the membership of the vector $\mathbf{x}^\mu$ in the sets H, F_0, S.*

4. SOLUTION ALGORITHM

The algorithm developed for the solution of the CP problem (1) consists of an initial step and an iterative procedure which additionally uses the sets E, P, and R. In the initial step, set $E = \emptyset$, $P = \emptyset$, $R = \emptyset$. Solve the P- problem to determine the vector $\mathbf{y}^0$ and the set $V_{\mathbf{y}^0}^\ell$, where $l=\sum_{j=1}^{m} y_j^0$. Set $P = P \cup \{\mathbf{y}^0\}$ and $E=E\cup V_{\mathbf{y}^0}^l\cup\{\mathbf{y}^0\}$. Index the vector $\mathbf{x}\in V_{\mathbf{y}^0}^l$ by

*The conditions $\mathbf{x}^\mu\in H$ and $\mathbf{x}^\mu\in F_0$ are checked by simple substitution in the constraints of the AE problem. The test of the inclusion $\mathbf{x}^\mu\in S$ is much more complicated, and this topic is considered separately in Appendix 2.

μ (μ = 1, ..., N) so that if $\mu_1 < \mu_2$, then $h(x^{\mu_1}) \leq h(x^{\mu_2})$. End of initial step. The iterative procedure selects the next vector x^δ in the order of increasing indices. It identifies to which of the states a, b, c, and d (see Sec. 3) the vector x^δ belongs and correspondingly determines one of the sets $V^l_{x^\delta}$, $Q^{l-1}_{x^\delta}$, $Q^{l+1}_{x^\delta}$ for x^δ. If the vector x^δ is in state c, then the set $V^{l^0}_{x^\delta}$ is determined, where $l^0 = \sum_{j=1}^{m} x_j^\delta$, and the subsets $V^{lp}_{x^\delta}$ and $V^{ld}_{x^\delta}$ are found. The set R is determined as the intersection of the sets $V^{lp}_{x^\delta}$ and $\{X \cap F_0\}$. The vectors from the set R are included in the set P and the minimum of the function $\Phi(x)$ is sought on these vectors. The resulting set of vectors is combined with the set E, the vectors are indexed by μ (μ = N + 1, ..., M) as described above, and the iterative procedure returns to the beginning. Let us describe the steps of the algorithms in detail.

Initial Step. Set P = ∅, E = ∅. Find the vector y^0 and determine its weight $l = \sum_{j=1} y_j^0$. Determine the set $V_{y^0}^l$. Set $P = P \cup \{y^0\}$. $\text{RECORD} = \Phi(y^0)$, $x_j^{CP} = y_j^0$, j = 1, ..., m, $E = \cup\{V_{y^0}^l \cup \{y^0\}\}$. RECORD stands for the best value so far. The vectors $x \in E \setminus \{y^0\}$ are indexed by μ (μ = 1, ..., N). Set δ = 1 and go to the beginning of the iterative procedure.

Iterative Procedure. Step 1. R = ∅.

Step 2. If $x^\delta \equiv H$, then go to step 6.

Step 3. If $x^\delta \equiv F_0$, then go to step 7.

Step 4. If $x^\delta \in P$, then go to step 8.

Step 5. Test x^δ for coordination by Algorithms 3 from Appendix 2. If $x^\delta \in S$, then $P = P \cup \{x^\delta\}$ and go to step 8, else go to step 9.

Step 6. For x^δ determine the set $Q^{l-1}_{x^\delta}$. Index the vectors $x \in Q^{l-1}_{x^\delta} \setminus \{E \cap Q^{l-1}_{x^\delta}\}$ by μ (μ = N + 1, ..., M). Set $E = E \cup Q^{l-1}_{x^\delta}$ and go to step 9.

Step 7. For x^δ determine the set $Q^{l+1}_{x^\delta}$. Index the vectors $x \in Q^{l+1}_{x^\delta} \setminus \{E \cap Q^{l+1}_{x^\delta}\}$ by μ (μ = N + 1, ..., M) and go to step 9.

Step 8. For x^δ determine the set $V^l_{x^\delta}$. Partition the set $V^l_{x^\delta}$ into subsets $V^{lp}_{x^\delta}$ and $V^{ld}_{x^\delta}$. Set $R = R \cup \{V^{lp}_{x^\delta} \cap \{F_0 \cap X\}\}$, $P = P \cup R$. Index the vectors $x \in V^l_{x^\delta} \setminus \{E \cap V^l_{x^\delta}\}$ by μ (μ = N + 1, ..., M), set $E = E \cup V^l_{x^\delta}$. Fix $x^0 = \operatorname{argmin} \Phi(x)$ over $x \in R \cup \{x^\delta\}$. If $\Phi(x^0) < \text{RECORD}$, then $\text{RECORD} = \Phi(x^0)$, $x_j^{CP} = x_j^0$, j = 1, ..., m, and go to step 9. Else go to step 9.

Step 9. Set N = M, δ = δ + 1. If δ > N, then go to step 10. Else go to step 1.

Step 10. RECORD is the solution of the CP problem (1)., and the vector x^{CP} is the sought coordinated plan.

We estimated the comlexity of the algorithm by counting the number of vectors tested for coordination, i.e., the number of vectors of the set F_0. The worst-case cardinality of F_0 does not exceed C_m^ℓ, where $l = p(y^0)$, $m = \dim y^0$. Therefore, the complexity of the algorithm is bounded by C_m^ℓ.

5. AN APPLICATION

The formulation of the problem and the mathematical model (1) were presented in a general form. When applying the results in real AS, the mathematical model may be substantially simplified by utilizing the specific features of the production divisions. Thus, in particular, the computer-aided subsystem for coordinated daily-shift operations planning of the stamping shop of the "Aktyubrentgen" industrial association minimizes the AE objective function writing it in the form $f(x, y) = \sum_{j=1}^{m} c_j y_j + \sum_{j=1}^{m} \alpha_j |x_j - y_j|$ where c_j are the divisional losses due to the auxiliary jobs for batch j. The components of the vector c are ordered as $c_1 \geq c_2 \geq \ldots \geq c_m$. The problems of the element and the center are knapsack problems. The constraints are identical for the center and the elements, as they describe the production capacity of the stamping shop. The constraints have a simple structure and an inequality of the type ≥. Therefore, the set F_0 can be replaced with the lower-level set $\underline{F}_0$ of the region H, which has the form

$$F_0=\{y\in H \mid Q_y^{l-1}\cap H=\phi\}.$$

The solution algorithm of the CP problem (1) also has been slightly modified. Thus, during the construction of the set S, the vectors x are indexed so that if $\mu_1 < \mu_2$, then $\sum_{j=1}^{m} c_j x_j^{\mu_1} \geqslant \sum_{j=1}^{m} c_j x_j^{\mu_2}$. Moreover, if $x^{\mu}\in H$, then the set $Q_{x\mu}^{l-1}$ is determined. These modifications simplify the application of the results for the determination of the daily-shift assignments of the stamping shop.

In addition to the mathematical system software, special programs and data bases were developed. The programs are different modules of the basic procedures, coded in FORTRAN IV under the OS-RV operating system. The data base was implemented using the tools of the SETOR-SM DBMS. The data base is stored on a SM 5400-00/12 magnetic disk. It consists of nine interlinked files and is maintained and supported by SETOR-SM DBMS tools.

The operating plans of the stamping shop are constructed by successively solving the following two problems:

1) the problem of determining a coordinated daily-shift plan;

2) the problem of optimal scheduling of the machines in the division.

Application of the results produced by the solution of the first problem (determining a coordinated daily-shift assignment for the stamping shop) has improved the continuity of operation of the machining, drilling, and assembly teams. The volume of work in process was reduced, and the finished product output accordingly increased by 3%. Note that the second problem is designed to reduce the volume of auxiliary jobs. The application of the results of its solution has made it possible to eliminate two setup positions, and the redundant workers are now employed as stampers. The stamping shop output accordingly increased by 6%. The economic impact of the computer-aided subsystem is 42 thousand rubles.

6. CONCLUSIONS

The CP problem has been formulated for a discrete AS and an appropriate mathematical model has been constructed. The properties of the set of coordinated plans have been identified and investigated. A solution algorithm for the CP problem has been developed and its complexity has been estimated. The application of the results in an instrument-building plant has been described.

Note that the model (1) may be used to describe the synthesis of a coordinated AS structure and the algorithm proposed in this paper is incorporated in the system software of the computer-aided production structure design system of the "Aktyubrentgen" industrial association. Moreover, the proposed algorithm can be used to solve coordinated planning problems with nonlinear objective functions, as well as ordinary discrete optimization problems. The corollaries of Theorem 1 were used as a basis for a direct method of solution of the CP problem (1), which differs from the method proposed in this paper in that it does not require preliminary identification of the set S and dispenses with a number of approximate algorithms.

APPENDIX 1

Proof of Lemma 1. Let $S \notin F_0$, i.e., there exists a vector $x\in S$ and $x\notin F_0$. Construct a new vector $x^1\in F_0$, by setting the j^*-th zero component of the vector x equal to 1, i.e., $x_{j^*}^1 = 1$ and $x_{j^*}=x_{j^*}^1-1$, $x_j=x_j^1$ $\forall j\neq j^*$. Since $x\in S$, then from the definition of the set S we have the following:

$$\sum_{j=1}^{m} c_j x_j \geqslant \sum_{j=1}^{m} c_j x_j^1-\chi(x, x^1)$$

or

$$\sum_{j=1}^{j^*-1} c_j x_j + c_{j^*}x_{j^*} + \sum_{j=j^*+1}^{m} c_j x_j \geqslant \sum_{j=1}^{j^*-1} c_j x_j^1 + c_{j^*}x_{j^*}^1 + \sum_{j=j^*+1}^{m} c_j x_j^1 - \chi(x, x^1).$$

Since $\forall j \neq j^*,\ x_j = x_j^1$, we have

$$c_{j^*} x_{j^*} \geqslant c_{j^*} x_{j^*}^1 - \alpha_{j^*} |x_{j^*} - x_{j^*}^1|.$$

Substitute $x_{j^*}^1 = 1$ and $x_{j^*} = 0$. As a result we obtain $c_{j^*} \leq \alpha_{j^*}$, and since $\forall j\ c_j > \alpha_j$, then $x \bar{\in} S$, a contradiction. Thus, $S \subseteq F_0$.

Proof of Theorem 1. Let $x^* \in S$, $x^0 \in V_{x^*}{}^{l p}$. Assume that $x^0 \bar{\in} S$. Then there exists a vector y^0 such that

$$\sum_{j=1}^{m} c_j x_j^0 < \sum_{j=1}^{m} (c_j y_j^0 - \alpha_j |x_j^0 - y_j^0|). \tag{A.1}$$

Denote $I^0 = \{j | x_j^0 = 1\}$, $I^* = \{j | x_j^* = 1\}$, $I^{y^0} = \{j | y_j^0 = 1\}$, $I_1^0 = I^0 \cup I^{y^0}$, $I_2^0 = I^0 \cap I^{y^0}$, $I_1^* = I^* \cup I^{y^0}$, $I_2^* = I^* \cap I^{y^0}$.

In order to prove the theorem, it suffices to show that the theorem holds for the following cases separately: 1) $y_{j_1}^0 = 1$, $y_{j_2}^0 = 0$; 2) $y_{j_1}^0 = 0$, $y_{j_2}^0 = 0$; 3) $y_{j_1}^0 = 1$; $y_{j_2}^0 = 1$; 4) $y_{j_1}^0 = 0$, $y_{j_2}^0 = 1$. The other components of the vector y_0 may be arbitrary.

1) By (A.1), since $x^* \in S$, we have

$$\sum_{j \in I^0} c_j < \sum_{j \in I^{y^0}} c_j - \sum_{j \in I_1^0 \setminus I_2^0} \alpha_j, \tag{A.2}$$

$$\sum_{j \in I^*} c_j \geqslant \sum_{j \in I^{y^0}} c_j - \sum_{j \in I_1^* \setminus I_2^*} \alpha_j. \tag{A.3}$$

Since $x^0 \in V_{x^*}{}^{l p}$, we have

$$\sum_{j \in I_1^0 \setminus I_2^0} \alpha_j = \sum_{j \in I_1^* \setminus I_2^*} \alpha_j + \alpha_{j_1} + \alpha_{j_2}.$$

We may therefore write

$$\sum_{j \in I^*} c_j \geqslant \sum_{j \in I^{y^0}} c_j - \sum_{j \in I_1^* \setminus I_2^*} \alpha_j > \sum_{j \in I^{y^0}} c_j - \sum_{j \in I_1 \setminus I_2^*} \alpha_j - \alpha_{j_1} - \alpha_{j_2} > \sum_{j \in I^0} c_j. \tag{A.4}$$

From $x^0 \in V_{x^*}{}^{l p}$ it also follows that $\sum_{j \in I^0} c_j \geqslant \sum_{j \in I^*} c_j$, which contradicts (A.4). The theorem is thus proved for case 1.

2) $y_{j_1}^0 = y_{j_2}^0 = 0$. Let $I_3^0 = I_1^0 \setminus I_2^0 \cup \{j_2\}$, $I_3^* = I_1^* \setminus I_2^* \cup \{j_1\}$. Clearly $I_3^0 = I_3^*$. Since $\alpha_{j_2} \geq \alpha_{j_1}$, then using (A.2) and (A.3) we write

$$\sum_{j \in I^*} c_j \geqslant \sum_{j \in I^{y^0}} c_j - \sum_{j \in I_3^*} \alpha_j - \alpha_{j_1} > \sum_{j \in I^{y^0}} c_j - \sum_{j \in I_3^0} \alpha_j - \alpha_{j_2} > \sum_{j \in I^0} c_j. \tag{A.5}$$

Since $x^0 \in V_{x^*}{}^{l p}$, then $\sum_{j \in I^0} c_j \geqslant \sum_{j \in I^*} c_j$. We thus have a contradiction in (A.5). The theorem is proved for the case 2.

3) $y_{j_1}^0 = y_{j_2}^0 = 1$. Denote $I_4^0 = I_1^0 \setminus (I_2^0 \cup \{j_1\})$, $I_4^* = I_1^* \setminus (I_2^* \cup \{j_2\})$. Clearly, $I_4^0 = I_4^*$. Using the condition $x^0 \in V_{x^*}{}^{l p}$ we rewrite the inequalities (A.2) and (A.3) in the form

$$\sum_{j \in I^{y^0}} c_j - \sum_{j \in I_4^0} \alpha_j - \alpha_{j_1} > \sum_{j \in I^0 \setminus \{j_2\}} c_j + c_{j_2}, \tag{A.6}$$

$$\sum_{j \in I^{y^0}} c_j - \sum_{j \in I_4^*} \alpha_j - \alpha_{j_2} \leqslant \sum_{j \in I^* \setminus \{j_1\}} c_j + c_{j_1}. \tag{A.7}$$

Multiplying (A.6) by −1 and adding the inequalities (A.6) and (A.7), we obtain

$$\alpha_{j_1}-\alpha_{j_2}<c_{j_1}-c_{j_2}. \tag{A.8}$$

Using (3), we have

$$\varepsilon(c_{j_2}-c_{j_1})>c_{j_2}-c_{j_1}.$$

This is a contradiction, which proves Theorem 1 for case 3.

4) $y_{j_1}{}^0 = 1$, $y_{j_2}{}^0 = 0$. Rewrite (A.2) in the form

$$\sum_{j\in I^0\setminus\{j_2\}} c_j + c_{j_2} < \sum_{j\in I^{y^0}\setminus\{j_2\}} c_j + c_{j_2} - \sum_{j\in I_1^0\setminus I_2^0} \alpha_j . \tag{A.9}$$

Since $x^0\in V_{x^{\cdot}}{}^{lp}$, we have $I^* = (I^0\setminus\{j_2\}) \cup \{j_1\}$. Add $c_{j_1} - c_{j_2}$ to both sides of (A.9). This gives

$$\sum_{j\in I^0\setminus\{j_2\}} c_j + c_{j_1} = \sum_{j\in I^*} c_j < \sum_{j\in I^{y^0}\setminus\{j_2\}} c_j + c_{j_1} - \sum_{j\in I_1^0\setminus I_2^0} \alpha_j . \tag{A.10}$$

From (A.10) it follows that $x^{\cdot}\bar{\in} S$. A contradiction. Theorem 1 is thus proved for case 4, which completes the proof. Q.E.D.

APPENDIX 2

Testing the Plan $x^{\cdot}$ for Coordination. By definition, a given plan $x^{\cdot}\in H\cap X$ is coordinated, i.e., $x^{\cdot}\in S$, if

$$\sum_{j=1}^{m} c_j x^{\cdot} \geqslant \sum_{j=1}^{m} [c_j y_j{}^0 - \alpha_j |x_j{}^{\cdot} - y_j{}^0|], \tag{A.11}$$

where $y^0=\operatorname{argmax}\left[\sum_{j=1}^{m}(c_j y_j - \alpha_j |x_j{}^{\cdot} - y_j|)\right]$ over $y\in H$. In order to find y^0 we need to solve the following problem:

$$\max_{y\in H}\left[\sum_{j=1}^{m} c_j y_j - \sum_{j\in J_1}\alpha_j(y_p - x_j{}^{\cdot}) - \sum_{j\in J_2}\alpha_j(x_j{}^{\cdot} - y_j)\right]$$

or

$$\max_{y\in H}\left[\sum_{j=1}^{m}\rho_j y_j + \sum_{j\in J_1}\alpha_j x_j{}^{\cdot} - \sum_{j\in J_2}\alpha_j x_j{}^{\cdot}\right], \tag{A.12}$$

where

$$\rho_j = \begin{cases} c_j+\alpha_j, & \text{if } j\in J_2, \\ c_j-\alpha_j & \text{if } j\in J_1. \end{cases}$$

Problem (A.12) is an ordinary 0-1 linear programming problem, and it can be solved by one of the standard techniques. In some cases, the following test of coordination is more efficient. Consider the system

$$\sum_{j=1}^{m}\rho_j y_j + \sum_{j\in J_1}\alpha_j x_j{}^{\cdot} - \sum_{j\in J_2}\alpha_j x_j{}^{\cdot} \geqslant \sum_{j=1}^{m} c_j x_j{}^{\cdot} + 1, \tag{A.13}$$

$$\sum_{j=1}^{m} b_{kj} y_j \leqslant b_k, \quad k=1,\ldots,K, \tag{A.14}$$

$$y_j\in\{0,\ 1\},\quad j=1,2,\ldots,m, \tag{A.15}$$

where (A.14), (A.15) define the set H of the AE problem. Denote by D_{x^*} the set defined by the system of inequalities (A.13)-(A.15) for the given plan x^*. If for the plan x^* the set $D_x^* = \emptyset$, then $x^* \in S$. In order to check the system (A.13)-(A.15) for consistency, we need upper and lower bounds on the weight of feasible vectors.

Let us determine the upper bound ℓ^* of the weight in any feasible solution of the system (A.13)-(A.15). Order for each k (k = 1, 2, ..., K) the coefficients b_{kj} (j = 1, ..., m) in nondecreasing order and count the maximum number ℓ^k of first elelments in the sequence $\{b_{k1}, \ldots, b_{km}\}$ whose sum does not exceed b_k. Set $\ell^* = \min \ell^k$ over $1 \le k \le K$. Using the definitions of J_1 and J_2, we obtain from (A.13)

$$\sum_{j=1}^{m} \rho_j y_j \geqslant \sum_{j\in J_2} (c_j+\alpha_j)+1=T.$$

Let us determine the lower bound ℓ^0 of the weight of any feasible solution of the system (A.13)-(A.15). To this end, order ρ_j (j = 1, ... m) in a nonincreasing sequence and count the minimum number of the first elements of the sequence $\{\rho_1, \ldots, \rho_m\}$ whose sum is greater than T. This determines the lower bound ℓ^0.

Obviously, if $\ell^0 > \ell^*$, then $D_x^* = \emptyset$.

Let $\ell^0 = \ell^*$. Then the check for consistency is carried out by a simple substitution in (A.13)-(A.15) of the numerical values of the vectors from the set W_ℓ, where $\ell = \ell^0 = i^*$, $W_l=\{y|p(y)=l\}$. The definition of the set W_ℓ for any ℓ is given below.

Let $\ell^0 = \ell^* - 1$. Then only vectors of weight ℓ^0 and ℓ^* are feasible, i.e., vectors from the sets W_{ℓ^0} and W_{ℓ^*}. System (A.13)-(A.15) is checked for consistency by Algorithm 1, whose basic idea is the following. Determine the set W_{ℓ^0} and identify the subsets $W_{\ell^0}{}^1$ and $W_{\ell^0}{}^2$, where

$$W_{l^0}{}^1=\{y\in W_{l^0}\,|\,h(y)<T,\ y\in H\},$$
$$W_{l^0}{}^2=\{y\in W_{l^0}\,|\,h(y)\geqslant T,\ y\in H\}.$$

If $W_{\ell^0}{}^2 \ne \emptyset$, then $D_{x^*}\neq\phi$ and $x^* \equiv S$. Otherwise, for all $y\in W_{l^0}{}^1$ determine the set $Q_y^{\ell+1}$. Let $W_{l^*}=\bigcup_{y\in W^1_{l^0}} Q_y^{l+1}$. Identify $W_{\ell^*}{}^2$. If $W_{\ell^*}{}^2 = \emptyset$, then $x^* \in S$, otherwise $x^* \equiv S$.

Let $\ell^0 < \ell^* - 1$. If $y \in D_{x^*}$, then $l^0 \leqslant p(y) \leqslant l^*$, i.e., only the vectors $y \in \bigcup_{l=l^0}^{l^*} W_l$ are feasible. The system (A.13)-(A.15) is checked for consistency by Algorithm 2, whose basic idea is the following. Initially set $l=\left]\frac{l^0+l^*}{2}\right[$. Determine the set W_ℓ. Then arbitrarily index the vectors of the set W_ℓ by γ (γ = 1, 2, ..., Γ_1). We obtain a sequence $\{y^1, \ldots, y^\gamma, \ldots, y^{\Gamma_1}\}$ whose states have not been checked. Assume that they are prohibited. The next vector y^γ may be in one of the following states.

1. Let $h(y^\gamma)<T,\ y^\gamma\in H$. Then for y^γ we obtain the set

$$W_{y^\gamma}{}^b=\left\{y\,|\,p(y)=\left]\frac{p(y^\gamma)+l^*}{2}\right[,\quad d(y^\gamma,y)=\left]\frac{p(y^\gamma)+l^*}{2}\right[-p(y^\gamma)\right\}.$$

To this end, we first determine the set $W_{\bar{\ell}}$, where $\bar{l}=\left]\frac{p(y^\gamma)+l^*}{2}\right[$. Then for $\forall y\in W_{\bar{l}}$ set $y_j=1,\ \forall j\in J_2^{y^\gamma}$ (where $J_2^{y^\gamma}=\{j\,|\,y_j^\gamma=1\}$). From $W_{\bar{\ell}}$ select vectors of weight $p(y)=\left]\frac{p(y^\gamma)+l^*}{2}\right[$. These vectors form the set $W_{y^\gamma}^b$.

2. Let $h(y^\gamma)>T,\ y^\gamma\equiv H$. Then for y^γ we obtain the set

$$W_{y^\gamma}{}^H=\left\{y\,|\,p(y)=\left]\frac{p(y^\gamma)+l^0}{2}\right[,\quad d(y^\gamma,y)=p(y^\gamma)-\left]\frac{p(y^\gamma)+l^0}{2}\right[\right\}.$$

To this end, first determine the set $W_{\bar{l}}$, where $\bar{l} = \left]\frac{p(y^{\gamma}) + l^0}{2}\right[$. Then for $\forall y \in W_{\bar{l}}$ take $y_j = 0,\ \forall j \in J_1^{y^{\gamma}}$ (where $J_1^{y^{\gamma}} = \{j \mid y_j^{\gamma} = 0\}$). From $W_{\bar{l}}$ select vectors of weight $p(y) = \left]\frac{p(y^{\gamma}) + l^0}{2}\right[$. They form the set $W^H_{y^{\gamma}}$.

3. Let $h(y^{\gamma}) \geqslant T,\ y^{\gamma} \in H$. Then $y^{\gamma} \in D_{x^*}$.

4. Let $h(y^{\gamma}) < T,\ y^{\gamma} \notin H$. Then y^{γ} is regarded as prohibited.

The new vectors are arbitrarily indexed by γ ($\gamma = \Gamma_1 + 1, \ldots, \bar{\Gamma}$).

The procedure of checking the system (A.13)-(A.15) for consistency is continued either until a vector $y^{\gamma} \in D_{x^*}$ is found or as long as the sequence $\{y^1, \ldots, y^{\bar{\Gamma}}\}$ contains at least one allowed vector. If no such vector exists, then $D_{x^*} = \emptyset$ and $x^* \in S$.

The set W_l, $0 \leq l \leq m$ is determined as follows. For the vector y^1 with the first l components equal to 1, determine the set $V_{y^1}{}^l$. Then for all $y \in V_{y^1}{}^l$ also determine the set $V_y{}^l$, and $W_l = \bigcup\limits_{y \in V^l_{y^1}} V^l_y$.

Before describing the algorithm that tests the vector x^* for coordination, we present a number of auxiliary conditions.

Assume that there exists a vector $x^0 \in V_{x^*}{}^{l^0} \cap H$, such that

$$c_{j_1} < c_{j_2} - \alpha_{j_1} - \alpha_{j_2}. \tag{A.16}$$

Then clearly, $x^* \notin S$.

Assume that $y^0 = \arg\max\limits_{y \in H} \sum\limits_{j=1}^{m} c_j y_j$ and

$$\sum_{j=1}^{m} c_j x_j^* < \sum_{j=1}^{m} (c_j y_j^0 - \alpha_j |x_j^* - y_j^0|). \tag{A.17}$$

From (A.11), $x^* \notin S$.

A given plan x^* is tested for coordination by Algorithm 3.

Step 1. Test condition (A.16) for x^*. If true, then $x^* \notin S$, and go to step 7. Else go to step 2.

Step 2. Test condition (A.17) for x^*. If true, then $x^* \notin S$, and go to step 7. Else go to step 3.

Step 3. For x^* construct the system of inequalities (A.13)-(A.15). Find l^0 and l^*. If $l^0 > l^*$, then $D_{x^*} = \emptyset$ and $x^* \in S$, go to step 7. Otherwise go to step 4.

Step 4. If $l^0 = l^*$, then go to step 4.1. Else go to step 5.

Step 4.1. Determine the set W_l ($l = l^0 = l^*$). If $W_l \cap D_{x^*} = \emptyset$, then $x^* \in S$ and go to step 7. Else $x^* \notin S$ and go to step 7.

Step 5. If $l^0 = l^* - 1$, then go to step 5.1. Else go to step 6.

Step 5.1. Apply Algorithm 1 to check $D_{x^*} \cap (W_{l^0} \cup W_{l^*}) = \emptyset$. If $D_{x^*} \cap (W_{l^0} \cup W_{l^*}) = \emptyset$, then $x^* \in S$ and go to step 7. Else $x^* \notin S$ and go to step 7.

Step 6. Apply Algorithm 2 to check $D_{x^*} \cap (\bigcup\limits_{l=l^0} W_l) = \emptyset$. If $D_{x^*} \cap (\bigcap\limits_{l=l^0}^{l^*} W_l)$ then $x^* \in S$ and go to step 7. Else $x^* \notin S$ and go to step 7.

Step 7. End.

LITERATURE CITED

1. V. N. Burkov and V. V. Kondrat'ev, Functioning Mechanisms of Organizations [in Russian], Nauka, Moscow (1981).

2. M. Mesarovic, D. Mako, and I. Takahara, Theory of Hierarchical Multilevel Systems, Academic Press, New York (1970).
3. V. A. Gorelik and A. F. Kononenko, Game-Theoretical Models of Decision Making in Ecological and Economic Systems [in Russian], Radio i Svyaz', Moscow (1982).
4. A. A. Ashimov, V. N. Burkov, B. A. Dzhaparov, and V. V. Kondrat'ev, Coordinated Management of Active Production Systems [in Russian], Nauka, Moscow (1986).
5. V. V. Kondrat'ev, "Current directions of research of organizational structures and functioning mechanisms," Izmeren., Kontrol', Avtomat., No. 2(54), 94-103 (1985).
6. V. N. Burkov, Elements of the Mathematical Theory of Active Systems [in Russian], Nauka, Moscow (1977).
7. K. S. Sagyngaliev and B. A. Dzhaparov, "Combinatorial coordinated planning problems," in: 8th All-Union Conf. on Control Problems, Abstracts of Papers [in Russian], Vol. 2, Inst. Probl. Upr., Moscow (1980), pp. 379-381.
8. K. S. Sagyngaliev, "The combinatorial problem of coordinated planning of the operation of one machine, I. Formulation and mathematical model of the problem," in: Topics of MIS Development for Technological Processes and Enterprises, KazPTI, Alma-Ata (1983), pp. 33-45.
9. K. S. Sagyngaliev, "The combinatorial problem of coordinated planning of the operation of one machine, II. Coordinated scheduling algorithms and conditions for mutually beneficial schedules," in: Topics of MIS Development for Technological Processes and Enterprises, KazPTI, Alma-Ata (1983), pp. 45-55.
10. R. G. Cassidy, M. J. L. Kirby, and W. M. Raike, "Efficient distribution of resources through three levels of government," Manag. Sci., 10, No. 8, 402-473 (1971).
11. V. S. Mikhalevich and V. L. Volkovich, Computational Methods for Analysis and Design of Complex Systems [in Russian], Nauka, Moscow (1982).
12. M. M. Medetov, R. D. Raimbekov, and K. S. Sagyngaliev, "Design of a coordinated production system," Avtomat. Telemekh., No. 4, 75-83 (1987).
13. K. S. Sagyngaliev, M. M. Medetov, and R. D. Raimbekov, "Design of a coordinated production structure," 4th All-Union Seminar on Methods of Synthesis and Development Planning of Large-Scale System Structures, Abstracts of Papers and Reports [in Russian], Inst. Probl. Upr., Moscow (1986), pp. 35-36.
14. K. S. Sagyngaliev, I. P. Shkurenko, M. I. Rubinshtein, and K. D. Raimbekov, "Solution of the scheduling problem for a multimachine production division," in: Control of Complex Technical Systems and Organizations [in Russian], KazPTI, Alma-Ata (1986), pp. 47-56.
15. K. S. Sagyngaliev, "Optimization methods in coordinated planning. II. An active system with linear elements," Avtomat. Telemekh., No. 10, 98-107 (1985).