

Logical foundations of experimental design for models of organizations are presented and a method of designing organizational experiments is described.

## 1. INTRODUCTION

Experimentation is one of the most common methods of research in natural sciences. Conducting experiments with organizations in order to study their functioning mechanism is a very time consuming and costly process. Experiments on models of organizations provide a technique for accelerating the research and reducing its costs. The experimental design theory suggests ways for further reducing the cost and time of the experiment and for increasing the accuracy and reliability of the results.

Experiments in organizations, which are also called active systems, differ from experiments in physics, chemistry, biology, and other sciences for which experimental design theory was originally developed. In this study, we consider experiments in the form of business names and simulation [1, 2]. A characteristic feature of these experiments is that the active behavior of people is one of the determining factors. As far as we know, experimental design theory so far has not been applied to the design of experiments in organizational research. We provide the logical foundations of experimental design in this field and describe a method of designing organizational experiments.

## 2. STATEMENT OF THE PROBLEM

The goal of the theory of active systems is to improve the functioning mechanism. The functioning mechanism of an organization is a set of rules (procedures, functions) that regulate the actions of the central authority and the subordinate elements in the process of functioning [3]. To this end, it is necessary to analyze the system and to use the analysis results to synthesize a functioning mechanism which is optimal by some criteria. These criteria are the formalized requirements that the active system is expected to satisfy, e.g., increasing the productivity of labor, coordinating lower-level and top-level interests, reliable reporting from the subordinate elements to central management, etc.

Formally, the problem of improvement of the functioning mechanism is stated in the following terms [3]. 1. Construct a mathematical model of the functioning active system. Use this model to investigate the actual functioning mechanism  $\hat{\Sigma}^0: \hat{\Sigma}^0 = \langle \hat{\phi}^0, \hat{\omega}^0, \hat{\pi}^0 \rangle$ , where  $\hat{\phi}^0$  is the system objective function,  $\hat{\omega}^0$  is the incentive system,  $\hat{\pi}^0$  is the planning law. This is the so-called analysis problem. Once solved, we can proceed to the synthesis problem. 2. Using the mathematical model of the system and the analysis results, choose the best model functioning mechanism  $\hat{\Sigma}^*$  from the set of all possible functioning mechanisms  $G^\Sigma$  satisfying the model of the system constraints. The sought functioning mechanism is obtained by maximizing the model functioning efficiency criterion  $\hat{K}(\langle \hat{\phi}, \hat{\omega}, \hat{\pi} \rangle)$ :

$$\hat{K}(\langle \hat{\phi}, \hat{\omega}, \hat{\pi} \rangle) \xrightarrow{G^\Sigma} \max.$$

Let  $\Sigma = \langle \phi, \omega, \pi \rangle$ , denote the functioning mechanism of the real system, as distinct from the functioning mechanism of the mathematical model of the system  $\hat{\Sigma}$ . The set of all system functioning mechanisms satisfying the constraints is denoted by  $G^\Sigma$ . The system efficiency criterion is  $K(\Sigma)$ . Let

$$\Sigma^* = \text{Arg max}_{G^\Sigma} K(\Sigma).$$

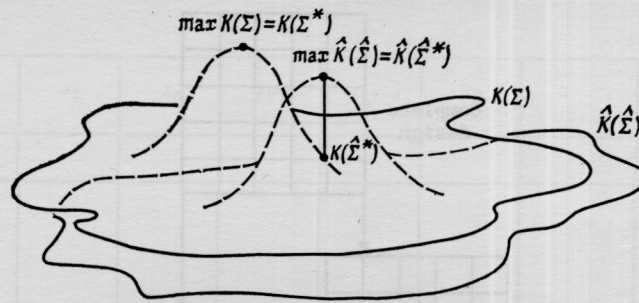


Fig. 1

Since the mathematical model of the system never achieves a completely faithful description of the real system,  $\hat{\Sigma}^*$  and  $\Sigma^*$  are not necessarily identical. This is demonstrated in Fig. 1. Therefore, before the functioning mechanism  $\hat{\Sigma}^*$  is adopted in the real system, it must be tested experimentally. Introduction of an untested functioning mechanism may result in losses that are incomparably higher than the cost of designing and conducting an appropriate experiment.

Experiments in organizations may be in the form of simulation, business game, or industrial experiment. In what follows, these terms are used only in the sense of an experiment in an organization or on a model of an organization.

In an industrial experiment, a system suggested by organization-theoretical research is introduced as a pilot project in one enterprise or in one industry. As a rule, any new introduction goes through a phase of industrial experiment. A different question is, whether the experiment is conducted in a literate way or not. If yes, the experiment will yield a wealth of information and suggest suitable adjustments, when required. The corresponding direction in experimental design theory is known as adaptive optimization, and appropriate design methods have been developed, such as the evolutionary design method.

Before launching an industrial experiment in an organization, however, it is advisable to experiment with a model of the organization, which is substantially simpler, cheaper, and less risky. It is the design of experiments on organizational models that constitutes the subject of our paper.

A business game is an experiment on a model of the system with people playing the role of active elements. In order to achieve a more or less reliable result in an experiment, it is necessary to conduct a series of trials and to evaluate the results statistically. But business games are usually time consuming, the more so since the players do not immediately grasp each new situation. It is therefore impossible to collect the required statistical material on the basis of business games only.

Fortunately, human players in a business game can be replaced by automata with memory. This leads to an experiment on a simultaneous model, which permits conducting all the necessary trials. But the simulation model alone is insufficient for making final conclusions, since the behavior of automata in any given situation does not necessarily match the much more complex behavior of people in the same situation. Automata are programmed on the basis of certain assumptions concerning the behavior of people in the given situation. These assumptions may be tested only in a business game.

Business game and simulation are therefore linked into a single experiment, which is designed as one whole. In this paper, we describe and demonstrate a method of designing such an experiment and processing the empirical data.

### 3. DESIGN OF THE EXPERIMENT

The design of an experiment on an organization model starts with the development of a suitable model and formulation of the experimental problem. For instance, the problem may be formulated as follows: investigate the dependence of the system efficiency criterion on system parameters and find the maximum of the efficiency criterion. Then the observed variable (or the response)  $y$  and the controlled variables (or factors)  $x_1, x_2, \dots, x_p$  are chosen for the particular experiment. In our example, the observed quantity is the system efficiency criterion.

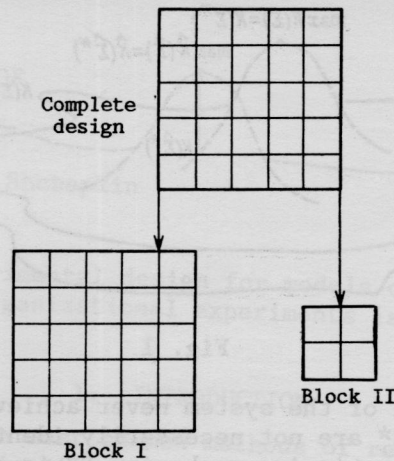


Fig. 2

Next a mathematical model of the experiment is constructed:

$$y = \beta_0 + \sum_{i=1}^p \beta_i x_i + \sum_{1 \leq i < j \leq p} \beta_{ij} x_i x_j + \dots + \beta_{12\dots p} x_1 x_2 \dots x_p \quad (1)$$

The model is a mapping of a point in the factor space  $(x_1, x_2, \dots, x_p)$  into the response space. In (1)  $\beta_i x_i$  is the effect of factor  $i$ ,  $\beta_{12\dots p} \prod_{i=1}^p x_i$  is the interaction effect of the factors  $x_1, x_2, \dots, x_p$ .

The experimental design is a collection of points in the factor space, with the response measured at each point. Schematically, this can be represented in the form of Table 1. The entries in each row are the factor levels. The numbers from 1 to  $N$  denote the different combinations of experimental conditions. As a rule, in order to collect sufficient statistical data, several trials should be conducted for each combination of experimental conditions.

The factor design corresponding to the chosen mathematical model of the experiment is applied to investigate the simulation model with automata players. As a result, we obtain a solution of the experimental problem (the maximum of the system efficiency criterion). But the original problem required finding this solution in the process of a business game, since a business game provides a more faithful representation of the real system than a simulation model does. The next stage therefore concentrates on experimental validation of the game with automata substituted for the business game with human players, i.e., checks that the response of the real business game coincides with the response of the automata game or, as we say in statistics, is statistically indistinguishable from it (the difference in the responses of the two games is not greater than the experimental error).

For validation purposes, the business game is run on some combinations of factor levels selected from the complete experimental design. Then the business game responses are compared with the responses of the automata game on the corresponding factor combinations. The business game and the automata game are essentially two blocks of the same experiment, with the trials distributed unevenly between the two blocks, as shown in Fig. 2 (the traditional experimental design employs uniform distribution of trials between the blocks). The validation of the automata game reduces to establishing absence of interblock effects, i.e., absence of significant difference between the mean responses of the different blocks.

The business game design is a randomly chosen fractional replication of the original design. The fractional replication may be  $1/2, 1/4, 1/8$ , etc., of the original factorial design, depending on the required accuracy of the business game and the allowed number of trials. The business game design is shown schematically in Table 2. Here  $i_1, i_2, \dots, i_L$  are the factor level combinations from the complete design included in the business game design.

In fractional replication, the effects represented by different terms in equation (1) are mixed and form groups of indistinguishable effects. We denote these groups by  $z_j$ . Then the mathematical model of the fractional replication design takes the form

TABLE 1

	$x_1$	$x_2$	$x_1x_2$	...	$x_p$	...	$\prod_{i=1}^p x_i$
1							
⋮							
N							

TABLE 2

	$x_1$	$x_2$	$x_1x_2$	...	$x_p$	...	$\prod_{i=1}^p x_i$
$i_1$							
⋮							
$i_l$							

TABLE 3

	$\hat{x}_1$	$x_2$	$\hat{x}_1x_2$	...	$x_p$	...	$\hat{x}_1 \prod_{i=2}^p x_i$
$i_1$							
⋮							
$i_L$							

$$y = \alpha_0 + \alpha_1 z_1 + \alpha_2 z_2 + \dots + \alpha_{L-1} z_{L-1} \tag{2}$$

The number of distinguishable effects in an experiment is equal to the number of factor combinations in the design less 1. Therefore, the number of effects that can be distinguished in a business game is  $L - 1$ , which is much less than the number of distinguishable effects in the simulation model. But the business game is not required to distinguish between all effects, since this is the task of the simulation model. It is only required to compare the experimental responses  $y$  and  $\hat{y}$  obtained in the business game and in the simulation experiment, respectively.

The factor level combinations induced in the business game are selected from the simulation experiment and entered in Table 3. Here the index 1 is assigned to the factor associated with active behavior. In the business game it represents  $x_1$  and in the simulation experiment  $\hat{x}_1$ . The mathematical model of the experiment conducted by the design of Table 3 has the form

$$\hat{y} = \hat{\alpha}_0 + \hat{\alpha}_1 z_1 + \hat{\alpha}_2 z_2 + \dots + \hat{\alpha}_{L-1} z_{L-1} \tag{3}$$

Let  $k$  be the index of a trial in a combination,  $K$  the number of trials in a combination,  $i_\ell$  the index of a combination,  $L$  the number of combinations,  $y_{i_\ell}^k$  the response in trial  $k$  of combination  $i_\ell$  in block I,  $\hat{y}_{i_\ell}^k$  the response in trial  $k$  of combination  $i_\ell$  in block II,  $r$  the

number of degrees of freedom of the experimental error. The responses of the two blocks shown in Tables 2 and 3 are compared by analysis of variance.

1. Compute the mean responses: general mean

$$\bar{y}_{\text{gen}} = \frac{1}{2LK} \sum_{k=1}^K \sum_{l=1}^L (\hat{y}_{i_l}^k + y_{i_l}^k),$$

block means

$$\bar{y}_{\text{BLI}} = \frac{1}{2K} \sum_{k=1}^K \sum_{l=1}^L \hat{y}_{i_l}^k, \quad \bar{y}_{\text{BLII}} = \frac{1}{2K} \sum_{k=1}^K \sum_{l=1}^L y_{i_l}^k,$$

row (combination) means

$$y_{\text{row}} = \frac{1}{2K} \sum_{k=1}^K (y_{i_l}^k + \hat{y}_{i_l}^k).$$

2. Compute the sums of squared deviations from the general mean. For each trial

$$s_{\text{gen}}^2 = \sum_{k=1}^K \sum_{l=1}^L (2\bar{y} - (y_{i_l}^k + \hat{y}_{i_l}^k))^2, \text{ for each block } s_{\text{BL}}^2 = LK(2\bar{y} - \bar{y}_{\text{BLI}} - \bar{y}_{\text{BLII}})^2 \text{ for each combination}$$

$$s_{\text{row}}^2 = 2K \sum_{l=1}^L (\bar{y} - \bar{y}_{\text{row}l})^2. \text{ The sum of squares of the experimental errors is computed from the}$$

basic analysis of variance equation  $s_{\text{err}}^2 = s_{\text{gen}}^2 - s_{\text{BL}}^2 - s_{\text{row}}^2$ , where the number of degrees for  $s_{\text{gen}}^2$   $2KL - 1$ , is  $s_{\text{BL}}^2$   $2 - 1 = 1$ , for  $s_{\text{row}}^2$   $L - 1$ , for  $s_{\text{err}}^2$   $(2KL - 1) - (2 - 1) - (L - 1) = 2(KL - 1) - (L - 1) = r$ .

The hypothesis of zero interblock effect is tested by the statistic  $F_{1,r} = s_{\text{BL}}^2 / (s_{\text{err}}^2 / r) = a$ .

The critical value of  $F_{1,r}$  is obtained from F-distribution tables with 1 and  $r$  degrees of freedom. Let the critical value for 75% confidence level be  $a_1$  and that for 90% confidence level  $a_2$ . Then if  $a > a_1$ , the hypothesis is rejected with probability of 75%, and if  $a > a_2$ , the hypothesis is rejected with probability of 90%. If  $a < a_1$ , then the hypothesis cannot be rejected, i.e., there is no interblock effect.

This experimental design method was developed under the assumption that the experimental response follows a normal or a nearly normal distribution. It is noted in [4, 5] that some authors have objected to the abuse of the normality assumption. In our case, this assumption is quite justified. If the variability of the outcomes is determined by a large number of factors, then the distribution of outcomes is most likely normal [6]. This assertion is based on the simple fact that the sum of infinitely many distributions is a normal distribution. In our case, the active behavior, and a fortiori the game outcome, are determined by a multitude of factors.

Comparing the coefficients in (2) and (3) we can infer if the behavioral hypotheses are correct, and if yes, then to what extent. If the behavioral hypotheses are incorrect, then the experiment described above may be used to improve the hypotheses. To this end, the experiment is conducted by a cyclic scheme. If the interblock effect is significant, but not very large, then in the next cycle we will not have to formulate new hypotheses, but only adjust the old hypotheses. The entire second-round experiment can be conducted by a simplified scheme.

The qualitative group composition must be taken into consideration when interpreting the experimental results in each stage. The simulation results may be inconsistent with the business game results if the players poorly grasp the situation and make irrational decisions. On the other hand, a game with a group of players who are familiar with the elements of the theory of active systems and thus behave more rationally may produce a better fit between the simulation experiment and the business game. The researcher has to decide if it is better to tune the automata or to train the human players. Bear in mind that the recommendations developed by active system theory are intended for experts.

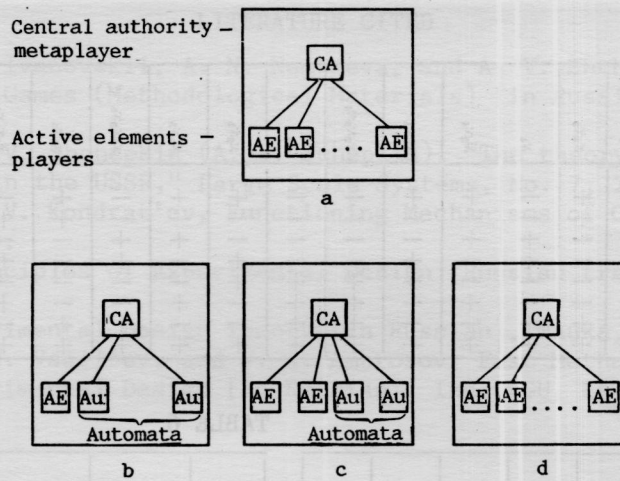


Fig. 3

The structural scheme of a business game (Fig. 3a) may have several variants:

- 1) one human player, all other players automata (Fig. 3b),
- 2) an intermediate variant (Fig. 3c),
- 3) all players human (Fig. 3d).

The first variant is the easiest to implement, since each game requires only one player. The last variant is the most difficult, since the number of players should be equal to the number of elements in the modeled system. Yet the first variant is unsuitable under the weak influence hypothesis. The researcher should choose that variant which is best suited to his task. In most cases, the intermediate variant is apparently the best, since it is free from the weaknesses of the first and requires fewer players than the last.

The proposed method thus may be applied to design an organizational experiment. The method is simple and may be used by a researcher without thorough knowledge of experimental design theory.

#### 4. EXAMPLE

The proposed method was applied to study the functioning efficiency of the model of a dynamic active system with long-sighted elements [3] under various planning laws.

The set of planning laws comprised five elements. It was required to select the planning law maximizing the system efficiency criterion. The efficiency criterion was the system objective function formed from the objective functions of the active elements. The problem was not solved analytically.

Five factors were identified which might influence the value of the efficiency criterion:

- 1) the planning law;
- 2) the number of elements in the system,
- 3) the output of the entire system during planned periods,
- 4) the production efficiency coefficients of the active elements,
- 5) the rate of growth of production efficiency coefficients.

The respective factors are designated  $x_1, x_2, x_3, x_4, x_5$ . The mathematical model of the complete factorial experiment, given the experimental problem, has the form

$$y = b_0 + \sum_{1 \leq i \leq 5} b_i x_i + \sum_{1 \leq i < j \leq 5} b_{ij} x_i x_j + \dots + b_{12345} x_1 x_2 x_3 x_4 x_5 + \varepsilon.$$

TABLE 4

	$x_2$	$x_3$	$x_2^2 x_3$	$x_2$	$x_2^2 x_4$	$x_2^2 x_5$	$x_2^2 x_3^2 x_4$	$x_5$	$x_2^2 x_5$	$x_3^2 x_5$	$x_2^2 x_3 x_5$	$x_4^2 x_5$	$x_2^2 x_4 x_5$	$x_3^2 x_4 x_5$	$x_2^2 x_3 x_4 x_5$
1	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
2	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
3	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
4	-	+	-	+	-	+	-	-	+	+	-	-	+	-	+
5	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
6	-	+	-	-	+	-	+	-	-	+	-	-	+	-	+
7	-	-	+	+	-	-	+	+	-	+	+	+	+	-	+
8	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

TABLE 5

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
1	+	-	+	+	-
2	+	+	-	-	+
3	-	-	+	+	-
4	-	+	-	-	+

TABLE 6

	$z_1$	$z_2$	$z_3 = z_1 z_2$
1	+	+	+
2	+	-	-
3	-	+	-
4	-	-	+

From physical considerations, significant effects are  $x_1$ ,  $x_2$ ,  $x_1 x_2$ ,  $x_3$ ,  $x_1 x_3$ ,  $x_4$ ,  $x_1 x_4$ ,  $x_5$ ,  $x_1 x_5$ ,  $x_1 x_4 x_5$ . The remaining effects are insignificant. The corresponding mathematical model is

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2 + b_3 x_3 + b_{13} x_1 x_3 + b_4 x_4 + b_{14} x_1 x_4 + b_5 x_5 + b_{15} x_1 x_5 + b_{145} x_1 x_4 x_5 + \varepsilon.$$

A specific feature of this experiment is that the factor  $x_1$  has five qualitative levels, while the remaining factors all have two quantitative levels, which are denoted +1 and -1.

The following experimental design was used. For each level of the factor  $x_1$ , the remaining four factors were held at the levels +1 and -1 according to one-half replication of the complete four-factor design shown in Table 4. The levels +1 and -1 are designated in the table by + and -, respectively. Thus, there are 40 factor combinations in the design. A simulation experiment was conducted using this design. The experimental response was the system objective function. As a result, we found the planning mechanism which maximized the response function and also established the form of the response surface, which enabled us to investigate the influence of all factors on the value of the system objective function.

When designing the business game, two levels were chosen for the factor  $x_1$ , one of which corresponded to the maximum response in the simulation experiment. For the factors  $x_2$ ,  $x_3$ ,  $x_4$ ,  $x_5$  we chose the combinations 4 and 5 from the one-half replication of Table 4, which constitute a one-eighth replication of the complete four-factor experiment.

The business game design thus included four combinations. The levels of the factors  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ ,  $x_5$  were set as shown in Table 5. The factors  $x_2$ ,  $x_3$ ,  $x_4$ ,  $x_5$  were mixed. The mixed effect of these factors is denoted by  $z_2$ . By  $z_1$  we denote the effect of the factor  $x_1$ . The interaction effect  $z_1 z_2$  is denoted by  $z_3$ . The business game design in this notation corresponds to a complete two-factorial design (Table 6). The corresponding mathematical model is

$$y = \alpha_0 + \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3.$$

The corresponding factor-level combinations were selected from the simulation experiment design. The estimates  $\hat{a}_0$ ,  $\hat{a}_1$ ,  $\hat{a}_2$ ,  $\hat{a}_3$  of the coefficients  $\hat{\alpha}_0$ ,  $\hat{\alpha}_1$ ,  $\hat{\alpha}_2$ ,  $\hat{\alpha}_3$  were obtained from the simulation experiment, and the estimates  $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$  of the coefficients  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$  were obtained from the business game. The values are equal as shown above.

The results did not support the hypothesis of significant difference between the estimates  $a_0$ ,  $a_1$ ,  $a_2$ ,  $a_3$  and  $\hat{a}_0$ ,  $\hat{a}_1$ ,  $\hat{a}_2$ ,  $\hat{a}_3$ . Thus, the simulation experiment produced a planning law maximizing the system efficiency criterion and the business game confirmed the simulation result.

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