COORDINATED OPERATIONS MANAGEMENT MECHANISMS FOR INDUSTRIAL SYSTEMS.

I. COORDINATED PLANNING IN PROCESS INDUSTRIES

A. A. Ashimov, V. N. Burkov,
B. A. Dzhaparov, and V. V. Kondrat'ev

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The article considers the problem of coordinated operations management whose goal is to ensure plan fulfillment in a two-level active system representing the process industry. The properties of incentive systems for the active divisions are investigated and constructive methods are proposed for determining the set of coordinated plans.

1. INTRODUCTION

The improvement of management mechanisms is a multifaceted problem which includes planning, incentives, socialist competition, etc. Experience shows that in order to improve the management of industrial systems, we have to jointly analyze all the management functions, such as the information base, planning, accounting, performance evaluation, control, and incentives. Focus on analysis and improvement of separate links in the management cycle fails to produce the full desired impact and in some cases may actually lead to totally unexpected results. For instance, development and installation of information and optimal planning systems which ignore performance evaluation and incentives may produce targets which are inconsistent with the incentive schemes used by the particular enterprise. As a result, the employees will differentiate between "profitable" and "unprofitable" jobs, exceeding the targets in the profitable jobs and failing to achieve the targets in the unprofitable ones. The manufacturing divisions may be driven to supply distorted information concerning their capacity, which in its turn will adversely affect the planning quality. In the final analysis, certain optimization procedures may be totally "rejected" by the organization.

Joint mathematical analysis of all the main functions and stages of industrial management required the development of a special apparatus and special models. These are primarily the topics of the theory of active systems [1, 2].

A topical current issue is the testing and implementation of the methods of active systems theory in actual economic organizations and the development of an appropriate methodology for this. The present series of papers examines the application of the methods of active systems theory in an actual industrial organization, i.e., for operations management of the principal industrial activity of a lead production plant (the V. I. Lenin Ust'-Kamenogorsk Lead-Zinc Combine).

In the framework of active systems theory, an industrial system is described by specifying the system structure, the state indices of the system elements, the model of the technology and the constraints, and the functioning mechanism of the system. The structure of an industrial system is specified by its structural elements and the interconnections or dependences between them. The state indices constitute a collection of phase variables which determine in a particular setting the state of the system as a whole and the states of its elements. The constraints on the phase variables are defined using the constraint model. The functioning mechanisms constitute a set of rules, functions, and assumptions which regulate the activity of all the functioning elements in the system.

The technology used to extract pure lead from lead ore concentrates is a complex sequence of industrial activities made up of continuous-process and discrete-batch stages. Of the wide range of operations management tasks in a lead production plant, we will only consider two typical examples which are characteristic not only of lead production plants but

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also of many industrial systems with continuous-process and discrete-batch production, respectively.

In Part I we consider the management of the charge preparation process, which is a version of the well-known mixing activity in process industries. In Parts II and III we consider operations management of a typical batch industry – the refining stage. For these industries, we construct coordinated operations management mechanisms, including coordinated planning procedures, performance evaluation methods, and divisional incentive schemes. The coordinated planning procedure may be regarded as further development of optimal planning methods to cases when the preferences of the active elements entrusted with plan performance must be taken into account. Constructively, these procedures are obtained by augmenting the optimal plnning problem with supplementary constraints, i.e., coordination conditions which ensure that the elements are assigned only such targets from the performance of which they stand to benefit under the given incentive scheme. The theory and the proof of effectiveness of these procedures are given in [2].

Our objective in this article is to determine one of the first examples of practical implementation of coordinated management methods in industrial systems and to describe a methodology of constructing coordinated operations management mechanisms in industrial systems using the methods of active systems theory.

2. DESCRIPTION OF THE TECHNOLOGICAL PROCESS

The pyrometallurgical technology of producing soft lead from lead concentrates [3] consists of the following sequential activities: charge preparation, sintering, smelting, and refining. These activities are respectively carried out in the sintering, smelting, and refining shops. A technological diagram of the plant producing soft lead from lead concentrates is shown in Fig. 1.

The feed (F) – lead concentrates, fluxes, semiprocessed ore – is delivered to the store (S) from outside sources. A separate store is provided for each type of feed. The charge preparation process (CP) involves the preparation of a mixture (wet charge, WC) of given composition from the available feed – lead concentrates and fluxes (limestone, iron ore, quartz). The charge is prepared by stockpiling in a special mixing area [4], which may accommodate several stockpiles at the same time. The returned sinters (RS) are also loaded into the stockpile: They are returned from the output of the sintering shop to be mixed with wet charge into what becomes sintering charge (SC). In order to ensure continuity of the lead production process, the stockpile must always contain sintering charge (of given composition) ready for loading.

The stockpile size may satisfy the sintering charge requirements of the lead production plant for two-to-three days. The sinter produced from the sintering charge is divided after the sintering stage (SIN) into usable sinters (US) and returned sinters (RS). The usable sinters are used as the feed for the smelting shop, which produces black (or crude) lead (BL). Sintering and smelting (SM) are both continuous processes.

The black lead from the smelting shop is continuously delivered to the collection kettle in the refining shop. When the collection kettle is full, its contents are pumped into the refining kettle for refining (R). The refining shop may also receive black lead from other plants in the form of cast blocks, which are loaded directly into the refining kettle. The refining technology for each batch of black lead involves sequential removal of various impurities: sulfur, tellurium, arsenic, antimony, tin, silver, gold, calcium, magnesium, bismuth. The degree of refining for each batch is determined by the required grade of current production. The refining process may be carried out simultaneously in several refining kettles, although a single distribution kettle is used to pour out the finished batch. The contents of the distribution kettle may be poured in the form of pigs or blocks, using one or two pouring machines. Unlike the sintering and smelting shops, the refining shop is characterized by discrete-batch operation.

The technology of producing soft lead (SL) from lead concentrates is thus a complex series of industrial activities which involve both continuous-process and discrete-batch operations.

3. THE PROBLEM OF COORDINATED OPERATIONS PLANNING OF CHARGE PREPARATION

The main task of the crushing and charge-making division is to prepare a charge of the required composition in a quantity which will be sufficient to enable the sintering shop to

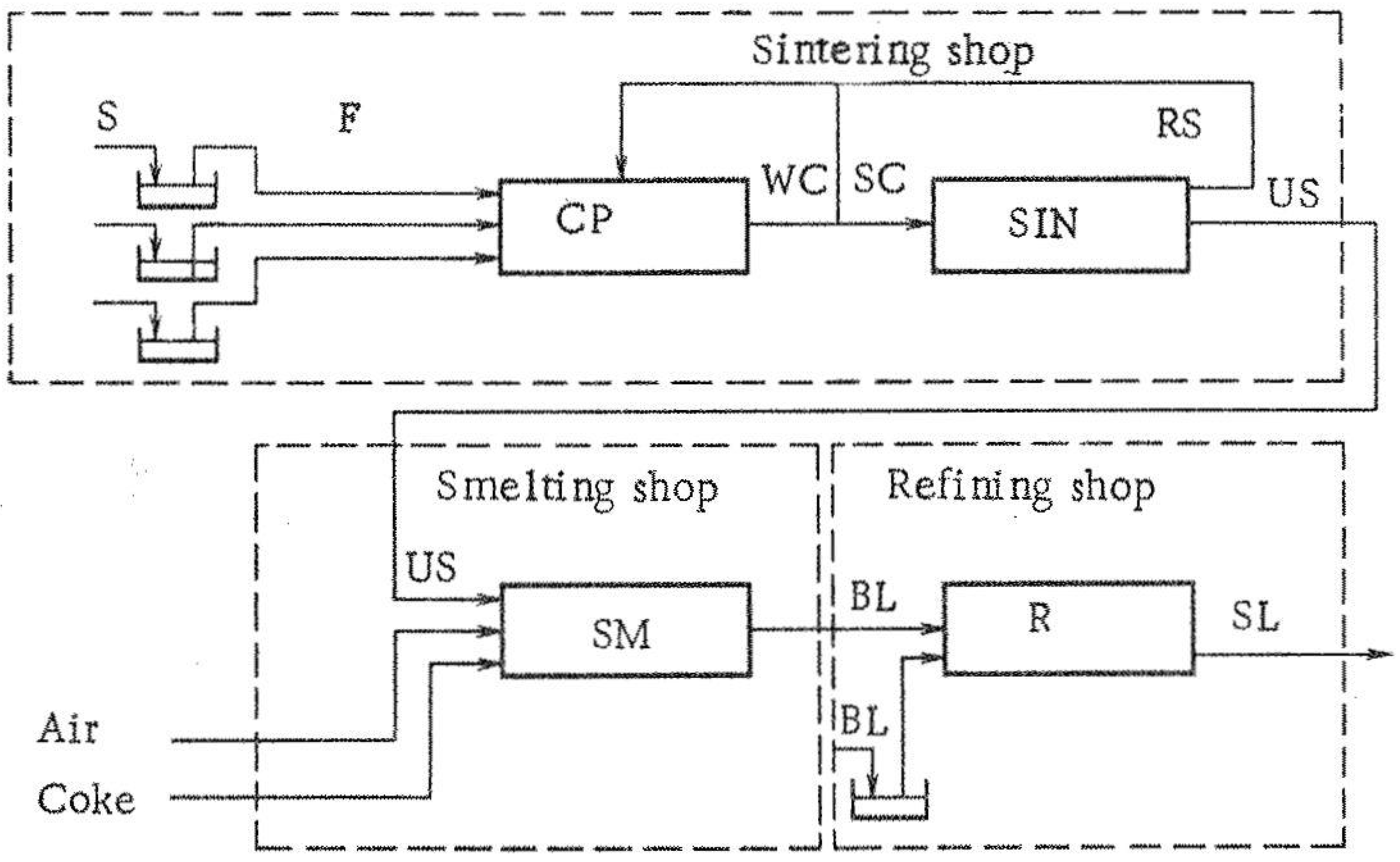

Fig. 1

meet its annual production target of usable sinters. Optimal charge preparation (mixing) methods and models have been studied in considerable detail in industrial management [5, 6]. There is, however, a big difference between setting a target and ensuring that the target is met.

The stockpiling plan is entrusted to charge-making crews who work in shifts. One stockpile is usually loaded over several shifts, which in its turn requires developing detailed shift targets for the crews. Note that the loading of the next stockpile begins before the previous stockpile has been completed and that during one shift a single crew is on duty whose "economic behavior" is determined by its economic payoff, which is a function of the shift target and the actual performance.

Thus the crushing and charge-making division may be represented as a two-level active system in which the headquarters is represented by the management of the sintering shop and the crushing and charge-making division, while the active elements are the charge-making crews (Cr) together with the stockpile loading activity (L). The "external environment" (EE) accounts for the interaction of the crushing and charge-making division with other processes (feed delivery, stockpile unloading, etc.). A structural diagram of the crushing and charge-making division is shown in Fig. 2.

The state of the system is defined by the state of the stockpile (ST), which by the end of the ν-th shift is described by the vector $\bar{y}_\nu = \{\bar{y}_{\nu j}\}$, where $\bar{y}_{\nu j}$ - is the volume of the concentrate j stockpiled since the beginning, $j \in J$, J is the set of concentrates, $\nu = \overline{1, V}$, V is the number of shifts required to load the stockpile. The state of an active element at the end of the ν-th shift is described by the state vector $y_\nu = \{y_{\nu j}\}$, where $y_{\nu j}$ is the volume of the concentrate j stockpiled in shift ν. The set of admissible states of the active element in the ν-th shift V_ν is determined by the technological constraints

$$0 \leq y_{\nu j} \leq q_{\nu j}, \quad j \in J; \quad \sum_{j \in J} y_{\nu j} \leq R; \quad \sum_{j \in J} t_j y_{\nu j} \leq T; \quad y_{\nu j} \leq R_j, \quad j \in J_1 \subset J, \tag{3.1}$$

where $q_{\nu j}$ is the quantity of the j-th concentrate in store in the ν-th shift, R is the standard total volume of feed loaded during one shift, t_j is the time to load a unit of concentrate j, T is the duration of one shift, R_j is the loading standard for concentrate j during one shift.

The headquarters sets the shift target $x_\nu = \{x_{\nu j}\}$ for each crew, taking into consideration the technological constraints, specifically the target mass $M = \sum_{j \in J} M_j$ and the target composition $\{M_j\}$ of the stockpile at the end of the planning period. The set X_ν of feasible production plans for the ν-th shift is thus constrained by the technological constraints

$$\begin{gathered} x_{\nu j} + \tilde{x}_{\nu j} = M_j - y_{(\nu-1)j}; \quad 0 \leq x_{\nu j} + \tilde{x}_{\nu j} \leq Q_{\nu j}; \quad \sum_{j \in J} \tilde{x}_{\nu j} \leq R(V-\nu); \\ \sum_{j \in J} t_j \tilde{x}_{\nu j} \leq T(V-\nu); \quad \tilde{x}_{\nu j} \leq R_j (V-\nu), \quad j \in J_1 \subset J, \end{gathered} \tag{3.2}$$

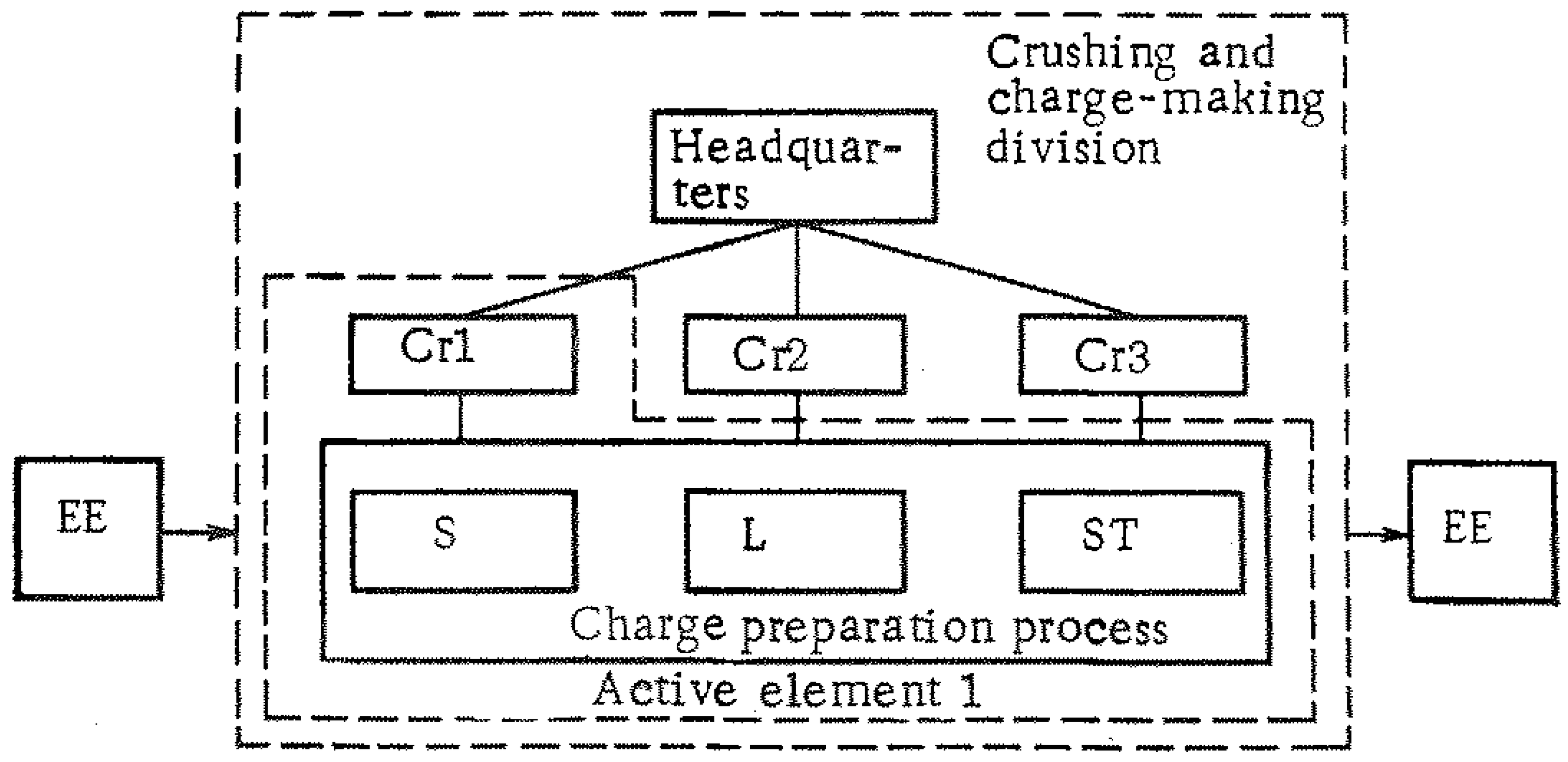

Fig. 2

where $x_{\nu j}$ is the loading target for concentrate j in shift ν, $\bar{x}_{\nu j}$ is the planned loading target for concentrate j in the remaining $(V - \nu)$ shifts, $Q_{\nu j}$ is the predicted delivery of j-th feed during V shifts, starting with shift ν.

The functioning of the system during the shift ν is described as follows: First headquarters assign a shift target $x_\nu \in X_\nu$ to the active element, after which the active element selects the state y_ν from the set Y_ν of admissible states, proceeding from economic considerations. This active behavior of the charge-making crew often results, in underperformance of the target x_ν, injecting an element of uncertainty into the management of the stockpiling process for the headquarters. The uncertainty can be eliminated by applying the methods of coordinated operations planning, i.e., by planning the concentrate loading targets of each shift while taking into consideration the actual atmosphere in the shop and the actual crew preferences. The incentive scheme for a crew in shift ν-th has the form

$$f(x_\nu, y_\nu) = h(\{c_j\}, y_\nu) - \chi(x_\nu, y_\nu), \tag{3.3}$$

where $h(\{c_j\}, y_\nu)$ is the basic pay of the crew, c_j is the unit cost of loading the j-th concentrate

$$\chi(x_\nu, y_\nu) = \begin{cases} \sigma > 0, & y_\nu \ngeqslant x_\nu, \\ (\alpha, x_\nu - y_\nu), & y_\nu \geqslant x_\nu, \ \alpha > 0 \end{cases} \tag{3.4}$$

is a penalty function, which essentially provides an incentive for exceeding the targets: The active element is penalized by the amount $\sigma > 0$ for failing to meet the target at least in one of the components and is encouraged to exceed the target. The incentive depends on the amount by which the target is exceeded and on the incentive coefficient $\alpha = \{\alpha_j\}$ representing the incentive rate for each component.

During system functioning, the headquarters presents various targets, some of which, e.g., reducing the charge cost, ensuring constant charge composition in the stockpiles, etc., may be incorporated in determining the optimal stockpile composition while others may be allowed for in operations planning. Thus, one of the factors in operations planning is ensuring uniform work load (in terms of cost) for the crews during the month.

The problem of coordinated operations planning involves determining the shift target x_ν for a crew so as to minimize the headquarters objective function:

$$\Phi(x_\nu, x_\nu) = \Big[B(\nu) + \mu - \sum_{j \in J} c_j x_{\nu j} \Big]^2 \to \min; \tag{3.5}$$

$$x_\nu \in X_\nu; \tag{3.6}$$

$$x_\nu \in S_\nu, \tag{3.7}$$

where $B(\nu)$ is the difference between the actual crew income and the average income of the previous shifts since the beginning of the month,

$$S_\nu = \{x_\nu \mid f(x_\nu, x_\nu) \geqslant f(x_\nu, y_\nu) \ \ \forall y_\nu \in Y_\nu\} \tag{3.8}$$

is the set of coordinated plans of the active element in shift ν X_ν is the set of feasible plans in shift ν defined by the constraints (3.1), (3.2), μ is the average income of the active element during one shift.

The problem of coordinated operations planning of charge preparation (3.5)-(3.8) may be considered as a mathematical programming problem whose solution can be obtained by standard optimization methods. These methods, however, assume canonical representation of the constraints in inequality form. The constraint (3.7) of the problem (3.5)-(3.7) is nonconstructive in this sense, and before we can proceed to solve the problem we have to describe the set S_ν of coordinated plans in constructive canonical form.

4. CONSTRUCTING THE SET OF COORDINATED PLANS

For simplicity, we omit the index ν. Common incentive systems use linear

$$h(c,y)=(c,y)=\sum_{j\in J} c_j y_j \tag{4.1}$$

or quadratic

$$h(c,y)={}^1/_2(Qy,y)+(d,y) \tag{4.2}$$

payoff functions with penalty functions of the form (3.4), where the sets X, Y are convex polyhedra defined by the system of linear inequalities $X = Y = \{y \mid Ay \leqq b,\ y \geqq 0,\ A = \{a_i\},\ a_i = \{a_{ij}\},\ b = \{b_i\},\ i = \overline{1, m},\ j = 1, n\}$.

1. Assume that the incentive system for the charge-making crews comprises a linear payoff function (4.1) and a penalty function (3.4).

THEOREM 1. Let $\chi(x, y) = \min\limits_{1\leqq k\leqq K} \{\chi_k(x, y)\}$, where $\chi_k(x, y)$ is also a penalty function corresponding to the set of coordinated plans

$$S_{\chi_k}=\{x \mid h(x)=\max_{y\in Y}(h(y)-\chi_k(x,y)\}.$$

Then $S_\chi=\bigcap\limits_{k=1}^{K} S_{\chi_k}$.

The proof of Theorem 1 is given in the Appendix.

It is easily seen that the penalty function (3.4) is obtained from two penalty functions

$$\chi_1(x,y)=\begin{cases} \infty & y\not\geqslant x,\\ (\alpha, x-y), & y\geqslant x,\ \alpha\geqslant 0\end{cases}$$

and

$$\chi_2(x,y)=\begin{cases} \sigma>0, & y\neq x,\\ 0, & y=x,\end{cases}$$

by applying the minimum operation, i.e., $\chi(x, y) = \min(\chi_1(x, y), \chi_2(x, y))$. Then by Theorem 1, the set S_χ of coordinated plans of the active element is formed as the intersection of two sets S_{χ_1} and S_{χ_2} of coordinated plans corresponding to the penalty functions $\chi_1(x, y)$ and $\chi_2(x, y)$, $S_\chi = S_{\chi_1} \cap S_{\chi_2}$.

Now, in order to construct the set of coordinated plans of a charge-making crew, it suffices to determine the sets S_{χ_1} and S_{χ_2}. The set S_{χ_2} of coordinated plans is constructed in the following way. First we solve the linear programming problem

$$h(y)=(c,y)\to\max, \tag{4.3}$$

$$Ay\leqslant b;\ y\geqslant 0. \tag{4.4}$$

Let y^* be an optimal solution of the problem (4.3), (4.4). Then the set S_{χ_2} is defined by the system of linear inequalities

$$Ay\leqslant b;\ y\geqslant 0;\ h(y)\geqslant h(y^*)-\sigma.$$

Now using the duality theory of linear programming, we can write the following system of linear inequalities to define the sets S_{χ_2} of coordinated plans: $(b, \lambda) = (c, z)$; $z \geqq 0$; $Az \leqq b$; $\lambda \geqq 0$; $y \geqq 0$; $Ay \leqq b$; $(c, y) \geqq (c, z) - \sigma$, where λ is the dual vector, $\lambda = \{\lambda_i\}$, z' is an auxiliary vector of the same dimension as y, z, $y \in Y$.

Let us now determine the set $S_{\chi 1}$ of coordinated plans. Let Y be a bounded set with nonempty interior, int $Y \neq \emptyset$. We will show that the set $S_{\chi 1}$ of coordinated plans is located at the boundary of the feasible set Y. Indeed, if $x \in \text{int } Y$, then $x \notin S_{\chi 1}$, since the element assigned this plan will prefer not to fulfill it: By performing $y = x + \varepsilon 1 \in Y$, $\varepsilon > 0$, where $1 = (1, 1, \ldots, 1)$ is a unit vector of the same dimension as x, the element will receive the payoff $f(x, y) = h(y) + \chi(x, y) = h(x + \varepsilon 1) + \alpha(y - x) = h(x + \varepsilon 1) + (\alpha, \varepsilon 1) = h(x) + (c + \alpha, \varepsilon 1) > h(x)$. Hence it is clear that the plan $x \in \text{int } Y$ is not coordinated, i.e., $x \notin S_{\chi 1}$.

We prove in the Appendix that the set $S_{\chi 1}$ of coordinated plans consists of subsets Γ_ℓ of the boundary of the set Y which are described by the system of linear inequalities of the form

$$(a_i, x) = b_i, \ i \in m(\Gamma_l); \ x \in Y,$$

where $m(\Gamma_\ell)$ is the subset of indexes (1, 2, ..., m) of the constraints (4.4) corresponding to the set Γ_ℓ, and the vectors a_i, $i \in m(\Gamma_\ell)$ of the constraint matrix (4.4) are such that in the n-dimensional Euclidean space E^n there is a vector β with positive components which can be decomposed in the vectors a_i, $i \in m(\Gamma_\ell)$, with nonnegative coefficients $\lambda_i : \beta = \sum_{i \in m(\Gamma_l)} \lambda_i a_i$, $\lambda_i \geqslant 0$, $\lambda_i \in E^1$.

We have thus elucidated the structure of the subset $S_{\chi 1}$ of coordinated plans for functioning mechanisms with incentive system based on linear payoff function. It has the form

$$S_{\chi_1} = \Gamma_1 \cup \Gamma_2 \cup \ldots \cup \Gamma_L,$$

where Γ_ℓ is a subset of the boundary of the feasible set Y satisfying a system of the form (3.1). Now, using Theorem 1, we find that the set S_χ of coordinated plans of the active element has the form

$$S_\chi = S_1 \cup S_2 \cup \ldots \cup S_L, \text{where } S_l = \Gamma_l \cap S_{\chi_2}.$$

<u>Example 1.</u> Let $h(y) = y_1 + 2y_2$; $\chi(x, y) = \begin{cases} 4, & y \not\geqslant x, \\ 0, & y \geqslant x, \end{cases}$ the set Y is defined by the system

$$0 \leqslant y_1 \leqslant 10, \ 0 \leqslant y_2 \leqslant 10, \ y_1 + 3y_2 \leqslant 33, \ 3y_1 + y_2 \leqslant 33, \ y_1 + y_2 \leqslant 15.$$

It is easily seen that for the optimal solution y^* the value of the objective function is $h(y^*) = 15$. The sets $S_{\chi 1}$, $S_{\chi 2}$, S_χ are shown in Fig. 3.

2. Let us now consider the incentive system $w = \langle h, \chi \rangle$ with payoff function (4.2), where the matrix Q is negative definite, and $Y = E^n$. In this case, the set of coordinated plans of the active element is again the intersection of two sets $S_\chi = S_{\chi 1} \cap S_{\chi 2}$.

The set $S_{\chi 2}$ is constructed as in the preceding section. We first find $h(y^*) = \max h(y)$ over $y \in E^n$. Then the set $S_{\chi 2}$ is defined by the inequality $h(x) \geq h(y^*) - \sigma = -{}^1/_2(Q^{-1}d, d) - \sigma$.

<u>THEOREM 2.</u> Let h(y) be an upward convex differentiable function, $\chi(x, y) = \max(b, y - x)$ over $b \in B$, B a closed convex set, $Y = E^n$. Then a necessary and sufficient condition for x to be a coordinated plan is $\nabla h(x) \in B$, where $\nabla h(x)$ is the gradient of the function h(x).

The proof of Theorem 2 is given in the Appendix. Theorems 1 and 2 were proved by the authors jointly with M. Z. Arslanov [7].

To find the set $S_{\chi 1}$, we use Theorem 2. To this end, we represent the function $\chi_1(x, y)$ in the form

$$\chi_1(x, y) = \max_{b \in B} (b, y - x),$$

where

$$B = \{b \mid b \in E^n, \ b \geqslant \alpha\}.$$

Hence it follows that $x \in S_{\chi 1}$ if and only if $Qx \geq \alpha$.

Finally, the set S_χ of coordinated plans is defined by the system of inequalities

$$Qx \geqslant \alpha, \ {}^1/_2(Qx, x) + (\alpha, x) \geqslant -{}^1/_2(Q^{-1}d, d) - \sigma.$$

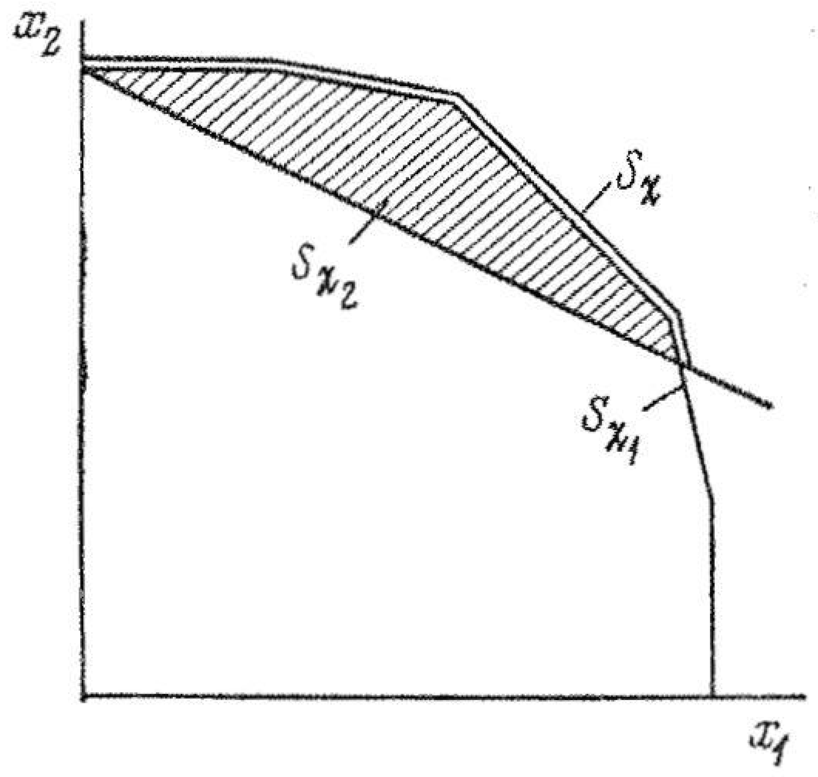

Fig. 3

Example 2. Let $h(y) = -{}^1/_2(y_1{}^2 + y_2{}^2)$, $y = \{y_1, y_2\}$, $x = \{x_1, x_2\}$.

$$\alpha=\{\alpha_1,\alpha_2\},\qquad \chi(x,y)=\begin{cases}\sigma, & y\not\geqslant x,\\ \alpha_1(x_1-y_1)+\alpha_2(x_2-y_2), & y\geqslant x.\end{cases}$$

Then the sets S_χ of coordinated plans are defined by the system

$$x_1^2/2+x_2^2/2\leqslant\sigma;\ x_1\geqslant\alpha_1,\ x_2\geqslant\alpha_2.$$

If $\alpha_1{}^2/2 + \alpha_2{}^2/2 > \sigma$, the set S_χ is empty. The condition ${}^1/_2(\alpha_1{}^2 + \alpha_2{}^2) \leqq \sigma$ is necessary and sufficient for nonemptiness of the set S_χ of coordinated plans.

5. SOLUTION OF THE COORDINATED PLANNING PROBLEM

Depending on the incentive system for the active elements and the objective function of headquarters, the problem (3.5)-(3.7) may have the following solutions.

Let the payoff function of an element be $h(y)=(c,y)=\sum_{j\in J} c_jy_j$, and let the penalty function be (3.4).

If the headquarters objective function $\Phi(x, x)$ is linear, the optimal coordinated plan is found as the solution of the following linear programming problem:

$$\Phi(x,x)\to\max\ \text{по}\ x\in\operatorname{conv}(S_\chi),$$

where $\operatorname{conv}(S_\chi)$ is the convex hull of the set S_χ. Here we use the fact that the optimal solution of a linear programming problem is one of the vertices of the feasible polyhedron.

If the headquarters objective function $\Phi(x, x)$ is nonlinear, we may separately solve L problems of the form

$$\Phi(x,x)\to\max,\ x\in S_l,$$

each with a polyhedral constraint set, and then compare all the L solutions in order to choose the best solution, $\ell = \overline{1, L}$.

Let us now solve the problem of coordinated operations planning of charge preparation (3.5)-(3.7). Applying the previous results, we obtain the following two problems (L = 2):

$$\Phi(x_\nu,x_\nu)\to\min\ \text{по}\ x_\nu\in X_\nu;\ x_\nu\in S_\nu{}^1, \tag{5.1}$$

$$\Phi(x_\nu,x_\nu)\to\min\ \text{по}\ x_\nu\in X_\nu;\ x_\nu\in S_\nu{}^2. \tag{5.2}$$

The set S_ν of coordinated plans for shift ν is the union of two sets $S_{\nu 1}$ and $S_{\nu 2}$: $S_\nu = S_{\nu 1} \cup S_{\nu 2}$, where $S_{\nu 1} = \Gamma_{\nu 1} \cap S_{\chi 1}$, $S_{\nu 2} = \Gamma_{\nu 2} \cap S_{\chi 2}$. The set $S_{\nu 1}$ is described by the system

$$\sum_{j\in J} x_{\nu j}=R;\qquad \sum_{j\in J} c_jx_{\nu j}\geqslant h^*-\sigma,$$

and the set $S_{\nu 2}$ is defined by the system

$$\sum_{j\in J} t_jx_{\nu j}=T,\qquad \sum_{j\in J} c_jx_{\nu j}\geqslant h^*-\sigma,$$

where h^* is the value of the payoff function of the active element for optimal solution of the linear programming problem $\sum_{j\in J} c_j y_{\nu j} \to \max$ over $y_\nu \in Y_\nu$. The quadratic programming problems (5.1), (5.2) are reduced by a standard transformation to a linear programming problem, which may be solved, say, by the simplex method. The best solution of these problems provides the solution of the original problem of coordinate operations planning of charge preparation.

We eould like to thank the following employees of the V. I. Lenin Ust'-Kamenogorsk Lead-Zinc Combine for their fruitful cooperation: A. S. Kulenov, L. V. Slobodkin, D. E. Estaev, A. T. Shabrin, and A. A. Beresnev.

APPENDIX

The following lemma determines the properties of the elements of the set $S_{\chi 1}$.

LEMMA. A feasible plan x is coordinated in the sense $x \in S_{\chi 1}$ if and only if the cone $E_+^n = \{z,\ z \geqq 0\}$ having its apex at the point x meets the set Y at the unique point x.

Proof of Lemma. Necessity. Let $x_0 \in S_{\chi 1}$. Then the cone of feasible directions $K(x_0)$ at this point may not contain a vector $p \in E^n$ with all nonnegative components. Otherwise, assigned the plan x, the element would prefer to perform $y = x + \varepsilon p \in Y$ with some $\varepsilon > 0$, since $f(x, y) = h(y) - \chi(x, y) = h(x + \varepsilon p) - \chi_1(x, \varepsilon p) = h(x) + h(\varepsilon p) + \varepsilon(\alpha, p) = h(x) + \varepsilon(c + \alpha, p) > h(x)$.

Therefore, by the separating hyperplane theorem, there is a vector β, $\beta \neq 0$ such that $(\beta, x) > (\beta, y)$, $x \in E_+^n/\{0\}$, $y \in K(x_0)$. Hence it follows that $(\beta, x) > 0$ and $\beta > 0$. Moreover $\forall y \in K(x_0): (\beta, y) \leqq 0$. Therefore, from the properties of convex cones we have

$$\beta \in -K^*(x_0) = \left\{ \sum_{i\in m(\Gamma_l)} \lambda_i a_i \,|\, (a_i, x_0) = b_i,\ \lambda_i \geqslant 0,\ i \in m(\Gamma_l) \right\}.$$

Sufficiency. Let the point x_0 be such that the cone of feasible directions $K(x_0)$ contains no vector with all nonnegative coordinates. Then $x_0 \in S_{\chi 1}$. Indeed, assume the contrary, $x_0 \notin S_{\chi 1}$. Then, $\exists y \in Y$

$$h(y) - \chi_1(x_0, y) > h(x_0).$$

Clearly, $y \geqq x_0$. But the vector $y - x_0$ is not in $K(x_0)$. Contradiction. QED.

Proof of Theorem 1. It suffices to prove the theorem for the case k = 2. The proof is in two stages.

Let $x \in S_\chi$; then $x \in S_{\chi 1} \cap S_{\chi 2}$. Indeed, from the definition of S_χ, we have for $\forall y \in Y$: $h(x) \geqq h(y) - \min(\chi_1(x, y), \chi_2(x, y))$. This inequality is equivalent to two inequalities: $h(y) - h(x) \leqq \chi_1{}^* x, y)$ for $\forall y \in Y$; $h(y) - h(x) \leqq \chi_2(x, y)$ for $\forall y \in Y$. But the first inequality is equivalent to $x \in S_{\chi 1}$, the second to $x \in S_{\chi 2}$. Therefore, indeed $x \in S_{\chi 1} \cap S_{\chi 2}$.

The proof that $x \in S_{\chi 1} \cap S_{\chi 2}$ implies $x \in S_\chi$ is developed along the same lines. QED.

Proof of Theorem 2. Sufficiency. Let $\nabla h(x) \in B$. Take the actual performance $y = x + \Delta x$; then $f(x, y) = h(x + \Delta x) - \chi(\Delta x) \leqq h(x) + (\Delta x, \nabla h(x)) - \chi(\Delta x)$ since the function h(y) is upward convex. From the definition of the penalty function, for all $b \in B$: $\chi(\Delta x) \geqq (b, \Delta x)$, including the case $b = \nabla h(x)$. Therefore $f(x, y) \leqq h(x) + (\Delta x, \nabla h(x)) - (\nabla h(x), \Delta x) = h(x) = f(x, x)$. Whence it follows that $x \in S_\chi$.

Necessity. Let $x \in S_\chi$, but $\nabla h(x) \notin B$. Since B is a closed convex set and $\nabla h(x) \notin b$, the separating hyperplane theorem [8] indicates that there exists y_0 such that for $\forall b \in B$ we have $(\nabla h(x), y_0) > (b, y_0)$ and therefore for y_0, $(\nabla h(x), y_0) > \max_{b\in B}(b, y_0)$.

Moreover, since B is a closed set and the linear function (b, y_0) is bounded from above, $\max(b, y_0)$ over $b \in B$ is attained. Thus, there exists $\alpha > 0$ such that $(\nabla h(x), y_0) - \alpha > \chi(y_0)$. Since the penalty function is homogeneous, we have $\chi(y)$ for $\forall y > 0$

$$(\nabla h(x),\ \varepsilon y_0) - \alpha\varepsilon > \chi(\varepsilon y_0).$$

Take $y = x + \varepsilon y_0$ for sufficiently small $\varepsilon > 0$. Then

$$f(x, y) = h(x + \varepsilon y_0) - \chi(\varepsilon y_0) = h(x) + (\nabla h(x),\ \varepsilon y_0) + o(\varepsilon) - \chi(\varepsilon y_0),$$

where $o(\varepsilon)$ is an infinitesimal of higher order than ε. By (A.1) we obtain $f(x, x + \varepsilon y_0) > h(x) + \alpha\varepsilon + o(\varepsilon)$. This implies that for some $\varepsilon_0 > 0$ we have $f(x, x + \varepsilon_0 y_0) > h(x)$, which contradicts $x \in S_X$. This completes the proof of necessity. QED.

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