ADAPTIVE FUNCTIONING MECHANISMS OF ACTIVE SYSTEMS

1. ACTIVE IDENTIFICATION AND PROGRESSIVE MECHANISMS

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We consider the analysis and synthesis of adaptive functioning mechanisms of a dynamic active system under incomplete information at the headquaters concerning the structure of the lower-level divisions. Conditions for exact structural identification of far-sighted active elements are established. Necessary and sufficient conditions are determined for optimality of adaptive functioning mechanisms.

1. INTRODUCTION

In [1] we analyzed the functioning of an active system under incomplete information at the headquarters assuming adaptive or lagged formation of data. We noted that the far-sightedness of the active elements must be taken into account for purposes of adaptive control of dynamic active systems under uncertainty.

In [2] a simple active element was considered, with stochastic constraints described by a stationary distribution function and the "actual achievement" being used as the basis for the planning. It was shown that in this case the active element sets an upper bound on the state variable, and as a result the active system performance is submaximal.

In [3] a two-level system was considered, consisting of the headquarters and an active subsystem; the subsystem objective function depends on the parameter α, $\alpha \in A$ where the true value (α_0) is not known to the headquarters. The conditions for the existence of solutions of the corresponding game-theoretic problems with adaptation were determined for finite A.

In this article we explore adaptive control of dynamic active systems under uncertainty. The main tool for elimination of the headquarters' uncertainty concerning the structure of the divisions is provided by the adaptive-control method. Adaptation and learning procedures are widely used in automatic control systems [4]. Adptive control in active systems requires the headquarters to use information about the plans and the states of the divisions in past periods in order to identify the divisional structure, to determine the current divisional plans, and to decide on controls [5]. In adaptive control, we distinguish between adaptive formation of data, when the structure refinement procedure is explicitly used (structure identification), and adaptive planning, when the plans for the current period are determined directly on the basis of the plans and the states in past period. The specific feature of adaptive control in active systems is that it must allow for the far-sightedness of the active elements. The information available to the active elements concerning the adaptive control procedures allows them to predict to a certain extent what the future managment decisions (plans) of the headquarters are going to be, depending on the state selected by the element "today." Therefore, the divisions may find it beneficial to restrict the set of feasible states in the current period (not to disclose the full divisional potential) in order to secure assignment of preferential plans in future periods. For instance, if adaptive control is based on identification, the result of identification may depend on the divisional activity, on the information available to the active element about the central identification and planning procedures, and on uncertainty eliminating actions of the element when selecting its state. The "nonclassical problems" arising in this context may be called "active-system identification" problems.

2. CONDITIONS OF EXACT IDENTIFICATION

Let us consider the conditions for exact structural identification of a farsighted active element (AE) described by m stage indicators $y = (y_1, \ldots, y_m)$ in the Euclidean vector space E^m. Let t denote the functioning period of the active system, t = 0, 1, ... By $Y_t(a)$

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we denote the state of feasible AE states, which is known to the headquarters up to a scalar parameter a, $a \in P \equiv [b, p] \subset R^1$. The true value of this parameter will be denoted by p, $p \in P$. We assume that $Y_t(a)$ is bounded, closed, and Hausdorff-continuous on P. If $Y_t(a)$ is also strictly monotone on P, i.e., $Y_t(a_1)) \supset Y_y(a_2)$, $W_t(a_1)) \cap W_t(a_2) = \emptyset$ for $a_1, a_2 \in P$; $a_1 > a_2$; W_t is the boundary of Y_t, then the following lemma holds.

LEMMA 1. For every $y_t \in Y_t(p)$, $p \in P$, there is a unique parameter $a \leqq p$ such that $y_t \in W_t(a)$.

Exact structural identification involves estimating the parameter $a = p$. As we assume that the headquarters objective function is a monotone function of the divisional indicators y_{tk}, $k = 1, m$, $t = 0, T$, and without loss of generality we may take it to be an increasing function. The boundary surface $W_t(p)$ determines the maximal (potentially atatainable) possibilities of the division, and we will therefore call p the divisional potential. Let us consider the following adaptive control scheme. In each period t, the headquarters obtains an estimate a_t of the parameter p from the conditions $y_t \in W_t(a_t)$, $a_t \geq a_{t-1}$ for the period $t + 1$, i.e., $x_{t+1} = \pi_{t+1}(a_t) \in Y_{t+1}(a_t)$. The procedure π_{t+1} may be called "actual-achievement planning." It is based on the following assumptions. First, the headquarters assumes that the division fully utilizes its potential in the current period (operating on the boundary of its feasible set); second, the headquarters is interested in ensuring full utilization of the divisional potential in the next period $t + 1$. A functioning mechanism which ensures full utilization of the AE potential (the division operates on the boundary of its feasible set, $y_t \in W_t(p)$, $t = 0, 1, \ldots$) is called progressive. We have to establish under what properties of the functioning mechanism the headquarters assumptions listed above are indeed true, i.e., we have to determine the sufficient conditions for a progressive functioning mechanism. We denote by $f(x, y)$ the divisional objective function, which is continuous in y, $y \in Y(p)$ for all $x \in Y(p)$.

Definition 1. A functioning mechanism is called weakly progressive if $\operatorname{Arg}\max_{t\ y_t} f(x_t, y_t) \in W_t(p)$ for all t, x_t, $p \in P$.

If the active element behavior is determined by maximization of the objective function $f(x_t, y_t)$, the structural identification problem for weakly progressive mechanisms is obviously solved fairly easily using the condition $y_t \in W_t(p)$. Let us now assume that the farsighted active element maximizes the efficiency criterion

$$w = w(\varphi_0, \varphi_1, \ldots, \varphi_T), \tag{1}$$

and w is an increasing function of the objective functions $\varphi_t = f(x_t, y_t)$, $t = \overline{0, T}$. We have to determine sufficient conditions for exact structural identification of the active elements for any monotone dependence of the divisional efficiency criterion on the objective functions in the current and future periods.

We introduce two definitions.

Definition 2. A functioning mechanism is called plan-progressive if $\max_{y \in Y_t(a_2)} f(\pi_t(a_2), y) \geqslant \max_{y \in Y_t(a_2)} f(\pi_t(a_1), y)$ for all $a_1, a_2 \in P$, $a_1 \leqq a_2$.

Definition 3. A functioning mechanism which is weakly progressive and plan-progressive is called strongly progressive.

The following theorem holds.

THEOREM 1. A strongly progressive functioning mechanism is a sufficient condition for exact structural identification of the active element.

All theorems are proved in the Appendix.

Remark. The strong progressiveness conditions are necessary in a certain sense. Specifically, if these conditions are violated, there is an efficiency criterion w and a value $p \in P$ such that the strategy $a_t = p$ is not optimal. Indeed, for some pair $a_1 < a_2$, a_1, $a_2 \in P$, let $t = \tau$

$$g_\tau(\pi_\tau(a_2), a_2) < g_\tau(\pi_\tau(a_1), a_2).$$

Take $p = a_2$, $w(\varphi_0, \varphi_1, \ldots, \varphi_T) = \varphi_\tau$. In this case the strategy $a_t = p$, $t = 0, 1, \ldots, T$ is no longer optimal, since the strategy $a_t = a_1$ for $t \leqq \tau - 1$, $a_t = a_2 = p$ for $t \geqq \tau$ ensures a higher vlaue of the efficiency criterion w.

Consider a more general case, when the divisional potential is determined by a vector parameter $(\bar{p})$, and not by a scalar parameter (p) as before. Theorem 1 is true in this case also if the sets $Y_t(\bar{a})$ are monotone in $\bar{a}$ (if $\bar{a}' \geq \bar{a}^2$, then $Y_t(\bar{a}^2) \subset Y_t(\bar{a}^1)$). Thus, under strongly progressive mechanisms, the division is willing to disclose its full potential.

The problem is thus purely technical – estimating the vector $\bar{p}$. Clearly, in order to obtain an exact estimator of $\bar{p}$, we need at least ℓ points on the surface $W_t(\bar{p})$, which requires observations of at least ℓ functioning periods. The sequence of states $y_0, y_1, \ldots, y_{l-1}$ should be such that the system of constraints

$$y_t \in W_t(\bar{p}), \quad t=0, 1, 2, \ldots, l-1$$

admits a unique solution. The construction of such a sequence (and thus of the corresponding plans) may require modifying the adaptive planning law (while preserving the progressiveness of the functioning mechanism), which in its turn may lead to temporary deterioration of control performance (for the duration of the exact identification).

3. THE OPTIMAL SYNTHESIS PROBLEM. STRONGLY PROGRESSIVE CONTROL LAWS

Let us consider a general statement of the synthesis problem for an optimal adaptive functioning mechanism of a two-level active system with N active elements. We denote by i the AE index, i = 1, N, superscripting with i all the variables introduced in Sec. 2. Thus, x_t^i, y_t^i, r_t^i, p_t^i are the plane vector, the state vector, the resource vector, and the potential of the i-th AE in period t. For simplicity, the entire sequence of all the admissible values of a particular index will be denoted by curly braces, without indicating the index bounds. The variable index will be omitted in this abbreviated sequence notation; thus $x_t = (x_t^1, \ldots, x_t^N) \equiv \{x_t^i\}$, $x = (x^1, \ldots, x^N) \equiv \{x^i\}$. Moreover, we assume that the indices take all the admissible values, unless otherwise qualified.

We assume that in each period the headquarters makes the first move (selects the functioning mechanism) and communicates its decision to the divisions. The adaptive functioning mechanism $\Sigma = \langle I, \pi, Q, f \rangle$ includes identification procedures $I = \{I_t^i\}$, planning procedures $\pi = \{\pi_t^i\}$, resource-allocation procedures $Q = \{Q_t^i\}$, and incentive procedures $f = \{f_t^i\}$, all of which are continuous functions

$$\begin{gathered} x_{t+1}^i = \pi_{t+1}^i(a_t^i), \quad r_t^i = Q_t^i(a_t^i), \quad a_t^i = I_t^i(x_t^i, y_t^i), \\ \varphi_t^i = f_t^i(x_t^i, y_t^i), \quad x_0 = x^0, \quad r_0 = r^0, \quad a = \{a_t^i\}. \end{gathered} \tag{2}$$

Here r_t^i are the resources assigned by the headquarters for period t + 1 at the end of period t. Without loss of generality, we may assume that the i-th AE at the moment t = 0 selects its state y^i depending on the available potential p, so as to maximize its objective function

$$w^i = w^i(\varphi_0^i, \varphi_1^i, \ldots, \varphi_r^i), \tag{3}$$

where w^i is a monotone increasing continuous function of its arguments. The functioning of the system thus may be treated as a game of N lower-level active elements. We denote

$$p_t^i = [b_t^i, c_t^i], \quad P = \prod_{t=0}^{T} \prod_{i=1}^{N} p_t^i, \quad Y(p) = \prod_{t=0}^{T} \prod_{i=1}^{N} Y_t^i(p_t^i),$$

$$\begin{gathered} R(\Sigma, p) = \{y = \{y_t^i\} \in Y(p) \mid w^i(f_0^i(x_0^i, y_0^i), \ldots, f_r^i(x_r^i, y_r^i)) \geqslant \\ \geqslant w^i(f_0^i(x_0^i, z_0^i), \ldots, f_r^i(x_r^i, z_r^i)) \quad \forall z = \{z_t^i\} \in Y(p)\}, \quad p \in P. \end{gathered}$$

It is easily shown that $R(\Sigma, p)$ is a compact set.

As the efficiency criterion of the functioning mechanism, we will use the guaranteed value of the headquarters continuous objective function $\psi(x, r, y)$:

$$K(\Sigma) = \min_{p \in P} \min_{y \in R(\Sigma, P)} \psi(x, r, y). \tag{4}$$

The optimal synthesis problem involves constructing a maximum-efficiency functioning mechanism [4].

Suppose that by using some identification procedure I we have determined, with some degree of accuracy, an estimator a of the true system potential p. This estimator may be

used to construct various planning and resource-allocation procedures ($x = \pi(a)$, $r = Q(a)$, $a \in P$) as well as the corresponding functioning mechanisms. We denote by $G_a^{\pi,Q}$ the class of adaptive functioning mechanisms with given adaptive planning procedures ($x = \pi(a)$) and given resource-allocation procedures ($r = Q(a)$) for the system potential estimator a. The following theorem will be useful in what follows.

THEOREM 2. The adaptive functioning mechanism Σ is optimal in the class $G_a^{\pi,Q}$ if

$$R(\Sigma, p) \in A(p) \quad \forall p \in P, \tag{5}$$

where

$$A(p) = \operatorname{Arg}\max_{y \in Y(p)} \psi(\pi(a), Q(a), y).$$

Note that in general condition (5) is not necessary.

Theorem 2 is useful for solving optimal synthesis problems of adaptive functioning mechanisms both when a structural identification procedure is given and when no explicit identification procedure is used, so that adaptive planning is based directly on past plans and states.

Let us consider an adaptive planning procedure with system potential preduction,

$$(\pi^*(a), Q^*(a)) = \arg\max_{(x,r) \in H(\Sigma)} \varphi(x, r), \tag{6}$$

where

$$\varphi(x, r) = \min_{y \in R(\Sigma, a)} \psi(x, r, y) \tag{7}$$

is the predicator of the guaranteed value of the headquarters objective function for given control parameters x, r and given potential predictor a, $H(\Sigma)$ is the set of admissible control parameters. Note that for $a = p$, the relationships (6), (7) specify an optimal planning procedure with system state prediction under complete information, as considered in [1].

We use $Z(\Sigma)$ to denote the set of adaptive planning and resource-allocation procedures ensuring progressiveness of the functioning mechanism Σ. If $H(\Sigma) \subset Z(\Sigma)$, i.e., the set of admissible control parameters $H(\Sigma)$ consists entirely of progressive planning and resource allocation procedures $Z(\Sigma)$, then $a = p$ (see Sec. 2). The efficiency of the adaptive functioning mechanism $\Sigma^*(p) \in G_p^{\pi,Q} = \langle \pi(p), Q(p), f \rangle$ with procedures (6), (7) is maximized in this case, since by (4) for every Σ

$$K(\Sigma) = \min_{p' \in P} \min_{y \in R(\Sigma, p')} \psi(x, r, y) \leqslant \min_{y \in R(\Sigma, p)} \psi(x, r, y) \leqslant$$
$$\leqslant \min_{y \in R(\Sigma, p)} \psi(\pi^*(p), Q^*(p), y) = K(\Sigma^*).$$

Thus, one of the approaches to the solution of the optimal synthesis problem of adaptive mechanisms involves constructing the set of progressive mechanisms which ensure exact structural identification of the active elements ($a = p$) and subsequently solving the problem of optimal planning with state prediction [1].

The optimal synthesis problem of an adaptive functioning mechanisms with efficiency criterion (4) thus reduces to the optimal synthesis problem of a progressive functioning mechanism and involves determining the maximum-efficiency progressive functioning mechanism.

Let us consider the case of a strongly progressive functioning mechanism. The planning procedure is represented by an arbitrary strongly progressive planning law (see Sec. 1). The optimal synthesis problem for a strongly progressive planning law (in short, SP-law) involves determining a maximum-efficiency SP-law. The conditions of plan-progressiveness for an independent AE have the form

$$\forall a_1, a_2, a_1, a_2 \in P: \quad g_t(\pi_t(a_1), a_2) \leqslant g_t(\pi_t(a_2), a_2).$$

Example. Consider an active system with a single AE, whose state will be denoted by $y = (y_1, y_2)$ (here and in what follows, the indices i, t are omitted for simplicity). The set of feasible states is

$$Y(p) = \{(y_1, y_2): y_1 + y_2 \leqslant p\}.$$

In each period, the headquarters objective function is

$$\psi(y)=c_1y_1+c_2y_2, \quad c_2>c_1,$$

and the divisional objective function is

$$f(\mathbf{x},\mathbf{y})=\begin{cases} -M, & \text{if } y_1<x_1 \text{ or } y_2<x_2, \\ k_1y_1+k_2y_2-\alpha(y_1+y_2-x_1-x_2), & y_1\geqslant x_1, \quad y_2\geqslant x_2, \end{cases}$$

where $k_1 > k_2 > 0$, $\alpha < k_2$, M is a sufficiently large positive number.

Let the estimator of the parameter p be $a \leqq p$. Then $x_1 + x_2 = a$, $y_1 + y_2 = p$. Clearly, the division selects $y_2 = x_2$, $y_1 = p - x_2$. We have

$$g(x, p)=k_1(p-x_2)-k_2x_2-\alpha(p-a).$$

Assuming that $x_2 = \pi_2(a)$ is a differentiable function of a, we write out the plan-progressiveness conditions in the form ($a_1 = a$, $a_2 = p$):

$$\frac{d\pi_2(a)}{da}\leqslant\frac{\alpha}{k_1+k_2}.$$

For $a = a_0$ we clearly have $\pi_2(a) = a_0$. The optimal planning SP-law thus has the form

$$\pi_1(p)=(p-a_0)\left(1-\frac{\alpha}{k_1+k_2}\right),$$

$$\pi_2(p)=a_0+\frac{\alpha}{k_1-k_2}(p-a_0).$$

Its efficiency, normalized by $\max\limits_{\mathbf{y}\in Y(p)}\psi(\mathbf{y})=c_2p$, is

$$K=\min_{a_0\leqslant p\leqslant p_{\max}}\left(c_1(p-a_0)\left(1-\frac{\alpha}{k_1+k_2}\right)+c_2\left(a_0+\frac{\alpha(p-a_0)}{k_1+k_2}\right)\right)\Big/ c_2p=1-\left(1-\frac{c_1}{c_2}\right)\left(1-\frac{\alpha}{k_1+k_2}\right)\left(1-\frac{\alpha}{p_{\max}}\right).$$

The solution of the synthesis problem of planning SP-laws in general is quite complex and requires further research.

Note that the planning procedure (6), (7) may prove to be highly efficient if we can identify the true system potential (p) with high degree of accuracy (i.e., $a \approx p$). Otherwise, this planning procedure may be far from optimal. In this case, the manifold of rational (in some sense) admissible planning procedures ($x = \pi(a)$) and resource-allocation procedures ($r = Q(a)$) for a given adaptive estimator a may be quite large. Necessary and sufficient optimality conditions for the corresponding functioning mechanism for arbitrary $\pi(a)$, $Q(a)$, i.e., in the class $G_a^{\pi,Q}$, are provided by (7). Now assume that (7) does not hold. It is useful to compare in terms of efficiency the mechanism ensuring that the plan is met $\mathbf{x}=\mathbf{y}$ and those that exceed the plan $\mathbf{y}\geqslant\mathbf{x}$.

Definition 4. A guaranteed correct functioning mechanism is a mechanism which ensures equality of states and plans of active elements for any system potential ($x = y \ \forall p \in P$).

LEMMA 2. A guaranteed correct mechanism (Σ_c) in the class $G^{\pi,Q}$ exists if and only if $x = \pi(a) \in Y(b)$, $b = \{b_t^i\}$.

In what follows we invariably assume that $\exists\Sigma_c \in G_a^{\pi,Q}$.

THEOREM 3. In the class $G^{\pi,Q}$, the mechanism Σ is no less efficient than the guaranteed correct mechanism Σ_c if and only if

$$R(\Sigma, p)\subset C(p, \Sigma) \quad \forall p\in P, \tag{8}$$

$$C(p, \Sigma)=\{\mathbf{y}\in Y(p)\,|\,\psi(\pi(a), Q(a), \mathbf{y})\geqslant\psi(\pi(a), Q(a), \pi(a))\}. \tag{9}$$

4. DISCUSSION

Adaptive control of evolving organizations must take into consideration the active behavior of their divisions, which is primarily manifested in varying degrees of utilization of the divisional potential (e.g., the production capacity) under conditions of incomplete information at the headquarters level [5].

The first group of problems arising in this context are associated with adaptive identification by the headquarters of the structure of the active elements. It is shown that a

sufficient condition for exact structural identification of the active elements is strong progressiveness of the functioning mechanism (Theorem 1). Note that the plan-progressivity of the functioning mechanism required in order to satisfy this condition has a fairly clear interpretation: As the target assignment of the active element is increased, its payoff should not decrease.

A number of problems are considered arising in the context of analysis and synthesis of optimal functioning mechanisms of evolving (growing) active systems. In case of exact structural identification of the active elements by the headquarters, the optimal synthesis problem reduces to solving the optimal planning problem with state prediction. On the other hand, in evolving active systems the headquarters may have difficulties constructing an optimal plan due to inadequacy of preductions for both objective and subjective reasons. An efficient strategy for the headquarters in this case is to utilize the available predictions in order to devise a realizable (balanced) plan with simultaneous synthesis of an incentive system which will encourage the divisions to utilize their internal resources fully (e.g., so as to exceed the plan). If there is a way to coordinate the objectives of the active elements with those of the headquarters for all the admissible values of the system potential, the corresponding functioning mechanism will be optimal in the class of adaptive functioning mechanisms utilizing these and only these predictions (Theorem 2). If such coordination is not feasible, the functioning mechanism is suboptimal in the given class. For this case we have derived necessary and sufficient conditions for the existence of adaptive functioning mechanisms which in general are more efficient than the correct mechanisms (Theorem 3).

APPENDIX

Proof of Theorem 1. Consider an arbitrary sequence of parameters $a_0, a_1, a_2, \ldots, a_{T-1}$ and the sequence corresponding to exact structural identification $a_t = p$, $t = 0, 1, 2, \ldots, T$.

We will show that

$$\max_{y \in Y_0(p)} f(x_0, y) \geqslant \max_{y \in Y_0(a_0)} f(x_0, y), \tag{A.1}$$

$$\max_{y \in Y_t(p)} f(q_t, y) \geqslant \max_{y \in Y_t(a_t)} f(x_t, y), \quad t = 1, 2, \ldots, T, \tag{A.2}$$

where $q_t = \pi_t(p)$, $x_t = \pi_t(a_{t-1})$.

The first inequality follows directly from the condition $Y(p) \supset Y(a_0)$. To prove (A.2), we denote $g_t(x(a_1), a_2) = \max_{y \in Y_t(a_2)} f(x(a_1), y)$. Then inequality (A.2) may be rewritten as

$$g_t(\pi_t(p), p) \geqslant g_t(\pi_t(a_{t-1}), a_t).$$

Since $a_{t-1} \leqq a_t \leqq p$, the plan-progressiveness and the growth of $Y_t(a)$ in a,

$$Y_t(a_2) \supset Y_t(a_1), \quad a_2 \geqslant a_1,$$

lead to

$$g_t(\pi_t(p), p) \geqslant g_t(\pi_t(a_t), p) \geqslant g_t(\pi_t(a_t) a_t) \geqslant g_t(\pi_t(a_{t-1}), a_t).$$

From (A.1), (A.2) and monotonicity of the efficiency criterion (1) in φ_i, $i = \overline{1, T}$, we obtain

$$w[g_0(x_0, p), g_1(q_1, p), \ldots, g_T(q_T, p)] \geqslant w[\varphi_0, \varphi_1, \ldots, \varphi_T].$$

The sequence y_t, $t = \overline{0, T}$, $y_t \in W_t(p)$ is thus optimal for the AE. In order to ensure that this sequence is reliable, it suffices to ensure that $\max_{y \in Y_0(p)} f(x_0, y)$ is attained on the boundary of the set $Y_0(p)$. Weak progressiveness of the functioning mechanism is sufficient for this. QED.

Proof of Theorem 2. In the class $G^{\pi, Q}$

$$K(\Sigma) \leqslant \min_{p \in P} \max_{y \in Y(p)} \psi(\pi(a), Q(a), y) = K_{\max}(\Sigma).$$

Using the conditions of the theorem, from the definition (4),

$$K(\Sigma) = \min_{p \in P} \min_{y \in R(\Sigma, p)} \psi(\pi(a), Q(a), y) \geqslant \min_{p \in P} \min_{y \in A(p)} \psi(\pi(a), Q(a), y) \geqslant K_{\max}(\Sigma),$$

whence follows sufficiency of the condition (5). QED.

Proof of the Lemma 2. Indeed, if $\pi(a) \in Y(b)$, then for $p = b$, $y^* = R(\Sigma, b) \subset Y(b)$, $x \neq y^*$, i.e., the mechanism is not guaranteed correct. If $b \leq p$, $Y(b) \subset Y(p)$ and $x = \pi(a) \in Y(p)$, $\forall p \in P$. Then selecting an incentive system with strong penalties for failure to meet the plan (f_{sp}), we obtain $y^* = R(\Sigma, p) = x$ $\forall p \in P$, $\Sigma_c = \langle \pi, Q, f_{sp} \rangle \in G^{\pi, Q}$. QED.

Proof of Theorem 3. For a guaranteed correct mechanism (Σ_c), $y^* = R(\Sigma, p) = x$ $\forall p \in P$ and $K(\Sigma_c) = \min_{p \in P} \psi(\pi(a), Q(a), x)$. But then, by (4), (8), (9),

$$K(\Sigma) = \min_{p \in P} \min_{y \in R(\Sigma, p)} \psi(\pi(a), Q(a), y) \geqslant$$
$$\geqslant \min_{p \in P} \min_{y \in C(p, \Sigma)} \psi(\pi(a), Q(a), y) = K(\Sigma_c),$$

which completes the proof of sufficiency.

Necessity is proved by contradiction. Let $K(\Sigma) > K(\Sigma_c)$ and let there be $p \in P$ such that $R(\Sigma, p) \subset C(p, \Sigma)$. Then by (9)

$$K(\Sigma) \leqslant \min_{p \in P} \min_{y \in R(\Sigma, p) \setminus C(p, \Sigma)} \psi(\pi(a), Q(a), y) \leqslant K(\Sigma_c),$$

a contradiction. QED.

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