MANAGEMENT IN ORGANIZATIONS: COOPERATIVE DECISION-MAKING

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Abstract

Game-theoretical model of the agents' coalitional interactions is proposed. General management problem is formulated. The incentive problems in radial and common-agency organization structures with coalitional interactions are solved. The processes of system structure changes because of coalitional interactions are analysed. The resource allocation problem in the system with coalitional interactions of agents is solved.

1 Introduction

There are two main motives that influence the economical behaviour of the individuals – the tendency to get the maximum of private benefit and unavoidable aspiration to cooperate with other people. The contradiction between these motives brings forth a conflict.

The mathematical basis of contemporary organizational management science is the game theory, which studies general models of conflict. Non-cooperative game theory concepts (e.g. Nash or Bayesian-Nash equilibrium) have long been used by the organizational management theory to formulate the models of agents' behaviour, thus leading to underestimation of agents' coalitional interactions in the management process.

A few publications have recently appeared [1, 2] that concentrate on coalitional interactions in an organization. Practically all of them are based on the model that permits asymmetric information and follow the Nash program [3] implying an explicit definition of a communication process between agents. The necessity to take into account complex communications occurring in real-life organizations makes the model of coalitional interactions so cumbersome that results can be achieved only in some particular cases.

At the same time, the classical cooperative game theory [4] could not propose a universally recognized game-solution concept, like Nash equilibrium. One more disadvantage of the cooperative game theory (CGT) is lack of ways available to describe incomplete (or asymmetric) information. That is why a CGT game model has been considered to be inconvenient for organizational management theory.

Although asymmetric information case is very important, there, nevertheless, exist a number of complete- and symmetric-information management problems that can be naturally and relatively simply described in terms of CGT.

We suggest a means to avoid the imperfections of CGT and use the concept of the game core to formulate a model of agents' coalitional interactions. This approach enables the principal to implement efficient management mechanisms that are valid under rather general assumptions of agents' parameters.

2 The model of coalitional interactions

Consider the system consisting of the principal and n agents. The payoff function $f_i(u, y_1,..., y_n, \theta)$ of i-th agent depends on the control action $u \in U$ of the principal, the actions $y_j \in A_j$ of all agents $j = 1,..., n$ and the state of nature $\theta \in \Omega$. The payoff function of the principal is $\Phi(u, y_1,..., y_n, \theta)$. The agents share the same information about the state of nature; therefore, they may exactly know it or may know some probability distribution or a set of possible states. For the sake of simplicity, we will assume the agents exactly know the state of nature, whereas the principal knows only Ω.

Denote $y := (y_1,..., y_n) \in A = \prod_{j=1}^{n} A_j$.

The principal chooses a feasible control action $u \in U$, and then the agents choose their actions given the control action of the principal. Then the principal learns the state of nature $\theta \in \Omega$ and all the agents and the principal get their payoffs.

Note that a set of the principal's control actions can be rather complex and may include "actions depending on agents' actions": $u = u(y)$, thus implementing Γ_2 meta-game [5].

A management problem for the principal is to find a feasible control action $u^* \in Arg \max_{u \in U} \min_{\theta \in \Omega} \min_{y \in P(u,\theta)} \Phi(u, y, \theta)$.

Here $P(u,\theta)$ is a set of possible agents' game outcomes, given the control action and the state of nature.

Unlike "classical" approach, which assumes $P(u,\theta)$ to be a set of Nash equilibrium points, we suppose $P(u,\theta)$ to be a set of cooperative game outcomes. Thus, we need to choose one of CGT game solution concepts as the basis.

To set the *cooperative game* (with transferable payoffs) is to define a set of players $N = \{1,...,n\}$ and a *characteristic function* $v: 2^N \to \Re^1$ that determines a payoff for every nonempty *coalition* $S \subseteq N$. Given the game in the normal form, one can calculate the value of a characteristic function for a coalition *S* as the value of two-person game with payoff functions $f_S(.) = \sum_{i\in S} f_i(x_S, x_{N\setminus S})$, $f_{N\setminus S}(.) = \sum_{i\in S} f_i(x_S, x_{N\setminus S})$ (where $x_S = (x_i)_{i\in S}$, $x_{N\setminus S} = (x_i)_{i\in N\setminus S}$) or an equilibrium payoff of a coalition *S* in this game.

A *payoff allocation* is any vector $x = (x_1, \ldots, x_n)$ where each component is interpreted as the utility payoff to player *i*. An allocation is *feasible* for coalition $S \subseteq N$ iff $\sum_{i\in S} x_i \le v(S)$. A coalition *can improve on* an allocation *x* iff $\sum_{i\in S} x_i < v(S)$. An allocation *x* belongs to the *core* of game (*N*, *v*) iff it is feasible for the *grand coalition N* and no coalition can improve on *x*. Thus, the core is a set of allocations that meet the following condition

$\sum_{i\in N} x_i = v(N)$, $\sum_{i\in S} x_i \ge v(S)$ for all $S \subseteq N$.

The mapping $\delta: 2^N \setminus \{N\} \to [0,1]$ is a *balanced cover* iff $\sum_{S: i\in S} \delta(S) = 1$ for any player $i \in N$.

The core is nonempty iff for any balanced cover δ $\sum_{S\subset N} \delta(S) v(S) \le v(N)$ [6].

A game with non-empty core is called *balanced*.

A cooperative game is *inessential* iff $v(S) = \sum_{i\in S} v(\{i\})$ for all $S \subseteq N$. The core of an inessential game includes only one payoff allocation $x_i = v(\{i\})$, $i \in N$.

If the only Nash equilibrium of agents' game is a strong Nash equilibrium [3] and we use an equilibrium payoff to calculate a characteristic function, then the cooperative game will be inessential.

Nonempty core ensures the stability of the grand coalition because no other coalition can guarantee better payoffs to its participants. Some other solution concepts (e.g. *NM*-solutions [4]) assume that the grand coalition will surely take place even in some unbalanced games, but we consider this assumption too optimistic.

The grand coalition of agents chooses the actions vector $y = (y_i)_{i\in N}$ to maximize its payoff function $f_N(u, y, \theta)$ given the control action *u* and the state of nature *θ*, so the solution $P(u,\theta) = Arg \max_y f_N(u, y, \theta)$.

If the game has an empty core, the analysis of coalitional interactions becomes much more complex, as different coalition structures of agents are possible. Therefore, a conservative principal must choose her actions leading to a balanced game and guaranteeing predictability of agents' behaviour.

3 Incentive mechanisms in radial structure under complete information

Consider the basic system discussed above. The principal's payoff function $\Phi(y) = H(y) - \sum_{i\in N} \sigma_i(y)$ is the difference of non-negative income *H*(*y*) and the total incentives that the principal pays to the agents. The payoff function of *i*-th agent $f_i(y) = \sigma_i(y) - c_i(y)$ is the difference of non-negative incentives received from the principal and non-negative cost function $c_i(y)$ depending on the action $y_j \in A_j = [0, +\infty)$ of each agent. The incentive problem for the principal is to choose *incentive scheme* (i.e. the vector $(\sigma_i(y))_{i\in N}$ of incentives functions) maximizing her payoff whereas agents maximize their payoffs given the incentive scheme. In [7] is shown that if the agents act non-cooperatively, under natural assumptions the optimal incentives schemes can be written as follows:

1. When the incentives of each agent may depend upon the actions of all the agents (i.e. $\sigma_i = \sigma_i(y)$), the incentive scheme

$$\sigma_i(y) = \begin{cases} c(y_i^*, y_{-i}) + \varepsilon_i, & y_i = y_i^* \\ 0, & y_i \ne y_i^* \end{cases}, \quad (1)$$

where $y_{-i} := (y_j)_{j\ne i}$, $y^* \in \operatorname{Arg}\max_{y\in A}[H(y) - \sum_{i\in N} c_i(y)]$ implements the actions' vector y^* as the only dominant strategies equilibrium and is *ε*-optimal for the principal.

2. When the incentives of every agent may depend only upon her action (i.e. $\sigma_i = \sigma_i(y_i)$), the incentive scheme

$$\sigma_i(y_i) = \begin{cases} c_i(y_i^*, y_{-i}^*) + \varepsilon_i, & y_i = y_i^* \\ 0, & y_i \ne y_i^* \end{cases}, \quad (2)$$

where $y^* \in \operatorname{Arg}\max_{y\in A}[H(y) - \sum_{i\in N} c_i(y)]$, implements the actions' vector y^* as the only Nash equilibrium and is *ε*-optimal for the principal.

3. If the principal observes some aggregated output $z = g(y)$ and her income depends only upon *z*, then the scheme

$$\sigma_i(z) = \begin{cases} c_i(y^*(z^*)) + \varepsilon_i, & z = z^* \\ 0, & z \ne z^* \end{cases}, \quad (3)$$

where $$\begin{cases} y^*(z^*) \in \operatorname{Arg}\min_{y: g(y) = z^*} \sum_{i\in N} c_i(y), \\ z^* \in \operatorname{Arg}\max[H(z) - \sum_{i\in N} \sigma_i(z)], \end{cases}$$

implements the actions' vector y^* as the only Nash equilibrium and is *ε*-optimal for the principal.

In the model under consideration any coalitional interaction is undesirable for the principal so she must ensure the result of non-cooperative game coincides with one of cooperative, i.e. the game is inessential.

In [8] is proven that the game is inessential (it has the only Nash equilibrium that is also strong one) for incentive schemes (1) and (3). For the incentive scheme (2) where the incentives of every agent can depend only upon her action, coalitional interactions can lead the agents to choose some vector of actions other than y^*, so additional payments from the principal are required and the efficiency of mechanism decreases.

4 Incentive mechanisms in common agency problem (complete information)

Common-agency systems, where several principals control the only agent, often arise in project-oriented and matrix organizational structures. One of the problems of such structures is the conflict of the project managers sharing the agent's manpower. Thus, it is of interest to obtain the conditions when this conflict does not take place. Consider the following incentive problem (Fig. 1).

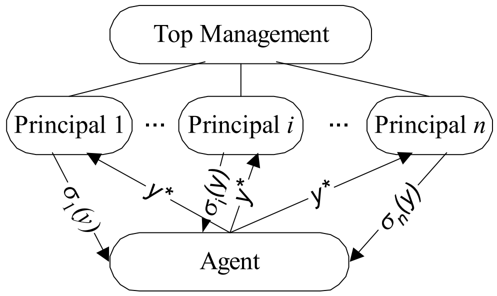

Figure 1. The model of common agency inventive problem

The principals have payoff functions

$\Phi_i(y) = H_i(y) - \sigma_i(y)$, $i \in N := \{1,\ldots,n\}$,

Here $H_i(y)$ is the i-th principal's income function and $\sigma_i(y)$ is non-negative incentives function from the i-th principal to the agent that depend on the agent's action $y \in A := \Re_+^m$.

The payoff function of the agent is $f(y) = \sum_{i \in N} \sigma_i(y) - c(y)$. Here $c(y)$ is non-negative cost function that increases on every component of the action vector.

To build a characteristic function of this game we should analyse the non-cooperative game of the principals' coalitions whose strategies are the incentives functions $\sigma_i(y)$. We will build a characteristic function using Nash equilibrium payoff of the principals' coalitions.

As shown in [9] the set of Nash equilibriums for the game of the principals' coalitions is rather vast. It includes both Pareto-optimal and non-Pareto-optimal outcomes.

We can make different assumptions about expected negotiation result for the game of coalitions:

1. If we suppose that every coalition S relies on *guaranteed* payoff among *all* the equilibriums then its characteristic function will be

$$v_G(S) := \max[\min_{y \in A} H_S(y); 0],\ S \subset N,$$

where $H_S(y) := \sum_{i \in S} H_i(y)$;

$$v_G(N) = \max_{y \in A}[H_N(y) - c(y)].$$

2. If the coalition S relies on *guaranteed* payoff among the *Pareto-optimal* equilibriums then its characteristic function is

$$v_{GP}(S) := G_S = \max_{y \in A}[H_S(y) - c(y)].$$

3. If coalition S relies on *mean* payoff among the *Pareto-optimal* equilibriums then its characteristic function is

$$v_{MP}(S) := 0{,}5(G_N + G_S - G_{N \backslash S}).$$

If appropriate cooperative game is balanced, we conclude that the principals can effectively resolve their conflict.

Theorem 1 [9]. If there exists some agent's action y, such that for all the principals $i \in N$ their incomes $H_i(y)$ are minimal, then the game $v_G(S)$ is balanced.

Theorem 2 [9]. The game $v_{GP}(S)$ is balanced for every separable agent's cost function $c(y) = \sum_{j=1}^{m} c_j(y_j)$ (where $c_j(.)$ – convex increasing functions and $y := (y_1,\ldots,y_m)$) and smooth concave increasing income functions $H_i(y)$.

$v_{MP}(S)$ is a constant-sum cooperative game so it is balanced only when the game of principals is inessential.

5 Coalitional interactions and system's structure changes

Consider the simple complete information incentives problem in the radial structure (see paragraph 3) with separable agents' cost functions. We assume that the principal's income function depends only on total output $Y := \sum_{i \in N} y_i$.

Given the set of the agents, the optimal inventive scheme for the principal is

$$\sigma_i(y) = \begin{cases} c_i(y_i^*), & y = y_i^* \\ 0, & y \neq y_i^* \end{cases},$$

$$y^* \in \operatorname{Arg}\max_{y \in A}[H(Y) - \sum_{i \in N} c_i(y)].$$

Depending on the transitory factors, (e.g. demand variations) the principal's income function will vary from period to period, and for the constant set of agents the set of agents will be redundant in some periods and it will be deficient in other periods. Thus, the principal must vary the set of agents in the system appropriately to achieve the best outcome.

If the principal for some reasons does not vary the set of the agents, the agents themselves can in an underhand way exclude redundant agent from the system. In this case all the agents gain, even excluded one, as he gets some discharge bonus.

To keep confidentiality the agents must reallocate the plan (and the wage) of the excluded agent among the remaining agents, thus such a structure change requires their coalitional interaction.

For an arbitrary coalition S of the agents, denote

$$C_S(Y) := \min_{\sum_{i \in S} z_i = Y} \sum_{i \in S} c_i(z_i)$$

– the least costs for the coalition to implement some total output Y, $Y_S = \sum_{i \in S} y_i^*$, $\sigma_S = \sum_{i \in S} \sigma_i$ – the total plan and remuneration for the coalition S respectively.

An action of the coalition S is to choose its subset to implement the plan Y_S with minimal costs. Therefore, the characteristic function of the game considered is

$$v(S) = \sigma_S - \min_{K \subseteq S} C_K(Y_S).$$

Theorem 3 [8]. If it pays to exclude some agent from every coalition, where he participates, then this agent will be excluded from the system and the remaining agents will distribute her plan to minimize total cost $C_{N\setminus\{e\}}(Y_N)$ of plan Y_N implementation.

This plans allocation is optimal for the principal if she prefers not to vary the set of the agents but is interested in costs minimization. Therefore, coalitional interaction of agents is advantageous for the principal in this model.

Above coalitional interaction concerns the case of redundant agent. One can also consider the case of deficient agents, where the agents can invite additional agents from outside of the system.

Consider an additional agent with cost function $c_a(y_a)$. The principal does not know about her existence. The coalition S of agents can gain from giving this additional agent a part of their plan Y_S if

$$\min_{y_a \geq 0}\left[C_S(Y_S - y_a) + c_a(y_a)\right] \leq C_S(Y_S) \ .$$

However, this condition can also be met for the coalition $N \setminus S$, so the conflict between these coalitions arises. The above investigation of the common agency problem shows that this conflict can result in overpayments to the additional agent and thus misallocation of system resources to outside.

However, such overpayments do not arise if additional agent joins the grand coalition N. Therefore, it is interesting to obtain some conditions for the grand coalition stability. These conditions are the conditions of non-empty core existence for this game.

Theorem 4 [8]. The cooperative game of the agents with concave cost functions is balanced, if the coalitions rely on the guaranteed Nash-equilibrium payoff when estimating their characteristic function (see $v_G(S)$ game in paragraph 4).

At that, the agents distribute their plans to minimize total costs of the principal's plan Y_N implementation.

6 Resource allocation

Consider the following resource allocation problem. The principal allocates some fixed amount R of resources among n agents. Concave utility functions $f_i(x_i)$ of the agents $i \in N = \{1,...,n\}$ depend upon the amount $x_i \geq 0$ of resource received.

The principal does not know exactly the agents' utility functions (he knows only that they are concave) but wants to allocate the resource to maximize the sum $\Phi(x_1,...,x_n) = \sum_{i \in N} f_i(x_i)$ of agents' utilities.

To obtain the information about the agents' utility functions the principal receives from them the messages $s_i \in [0, R]$, i.e. the amounts of the resource the agent prefers to receive. Then the principal allocates the resource according to the resource allocation mechanism $x_i = \pi_i(s_1, \ldots, s_n),\ i \in N$.

We will analyse continuous direct-priority resource allocation mechanisms (where the amount of resource, received by i-th agent, continuously increases as her message s_i increases given the messages of the other agents).

First, consider non-cooperative resource allocation model without any coalitional interactions. In this case, the agents are supposed to choose their Nash-equilibrium messages. Under the natural assumption of resource deficit, the agents start manipulating their messages to receive more resource.

The analysis of this non-cooperative game shows [10] that one can divide the agents in two groups: the dictators and the non-dictators. The dictators fulfil their resource needs (i.e. receive the amount of resource that maximizes their utility function). The non-dictators reveal maximal feasible messages to receive as more resource as they can while the dictators send only such messages, that allow them to obtain just as much resource as they need.

An efficiency of the resource allocation mechanism is a ratio of actual principal's payoff value $\Phi(x_1,...,x_n)$ to its maximal value $\max_{\sum_{i \in N} x_i = R} \sum_{i \in N} f_i(x_i)$.

A guaranteed efficiency K_0 of an allocation mechanism is its efficiency for the worst possible agents' payoff functions profile.

Denote $r_i := \arg\max_{x_i \geq 0} f_i(x_i)$ – the optimal amount of resource for the i-th agent.

Theorem 5 [8]. Consider an arbitrary direct priority continuous resource allocation mechanism. If the principal knows that $r_i \leq \overline{r_i}$, $i \in N$ then $K_0 = \min_{i \in N}\{x_i(\overline{r_1},...,\overline{r_n})\}/R$. Another information (like $\underline{r_i} \leq r_i$, $i \in N$) does not change the guaranteed efficiency $K_0 = \min_{i \in N}\{x_i(R,...,R)\}/R$.

The necessary condition of the informational coalitions formation (the weakest type of coalitional interaction) is the possibility for the agents to choose their messages cooperatively. The other coalitional interactions can be divided in two types:

1. The resource reallocation among the agents;
2. The payoff transfers among the agents.

Consecutively allowing these types of the coalitional interactions, we obtain the following models:

1. No resource reallocation, no payoff transfers (only cooperative decision-making is permitted);
2. The resource reallocation is permitted, but not the payoff transfers;
3. Both the resource reallocation and the payoff transfers are permitted;

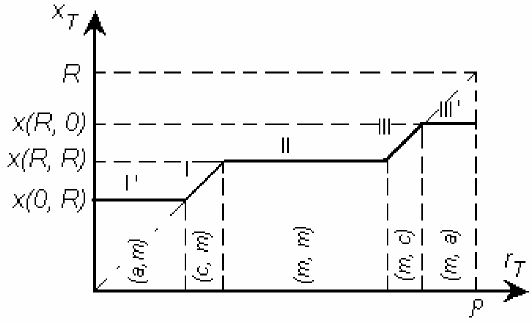

Figure 2. The dependence of the resource volume x_T received by the coalition $T \subseteq N$ in the Nash equilibrium upon its resource requirements r_T (here $\rho := r_N$)

Theorem 6. For the models 1 and 2 the coalitional interaction can not change the non-cooperative outcome of the agents' game.

Thus, the only model where the real cooperation can arise is the third one as the abilities of the cooperation in this model are the highest ones.

The characteristic function value for coalition T is determined by the coalition Nash-equilibrium payoff in its game with the coalition $N \backslash T$.

As shown in [8] the characteristic function is

$$v(T) = \max_{\sum_{i \in T} y_i = x_T(r_T)} \sum_{i \in T} f_i(y_i),$$

where $r_T := \sum_{i \in T} r_i$ is the resource requirement of the coalition T and the dependence $x_T(r_T)$ is shown on Fig. 2.

If this game of the agents is balanced then the principal can be sure that rational agent will join the grand coalition. The payoff function of the grand coalition coincides in this game with the principal's payoff function. Therefore, the grand coalition will redistribute the resource optimally among its participants and the efficiency of each resource allocation mechanism that leads to the balanced game is maximal.

Theorem 7 [8]. If for every balanced cover δ_T $\sum_{T \subset N} \delta_T x_T(r_T, T) \le R$ then the game of the agents is balanced.

The verification of this theorem conditions shows that for every direct-priority resource allocation mechanism there exists such agents' utilities profile that the game is not balanced. Nevertheless, if the principal has additional information about agents' resource requirements (e.g. the vector $(r_i)_{i \in N}$ of optimal resource allocations belongs to some set $L \subseteq \Re^n_+$) sometimes she can be sure that the game is balanced.

Corollary 1 [8]. If for every coalition T its resource requirements r_T belong to the zone II (see Fig. 2) then the game is balanced.

Corollary 2 [8]. If there exists an agent k such that for all coalitions, including this agent, their resource requirements belong to the zone III (se Fig. 2) and otherwise – to the zone I, then the game is balanced.

In other words, the game is balanced if all the agents have relatively great requirements or there is the only agent, whose requirements are much greater than the requirements of the other agents.

Note that the resource requirements in these conditions are bounded below. As theorem 5 shows, such restrictions do not change the guaranteed efficiency of resource allocation mechanism in non-cooperative model. Yet, this information proves to be extremely useful with cooperative approach.

Thus, one can see that the continuous direct-priority mechanism cannot ensure the game is balanced for all agents' payoff function profiles. Therefore, to obtain the surely balanced game one has to consider other classes of resource allocation mechanisms.

Corollary 3 [8]. The game of the agents is always balanced for the constant-priority mechanism $x_i = \frac{A_i R}{\sum_{j \in N} A_j}$, where every agent receives the constant volume of the resource irrespective of her resource request.

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