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INCENTIVES IN TEAMS UNDER FUZZY UNCERTAINTY

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Abstract

Game theoretical incentive problem in the framework of the fuzzy principal-agent model is considered and optimal incentive schemes are constructed. The guaranteed efficiency of management under fuzzy uncertainty is shown to be lower than the deterministic one and decreased with the growth of uncertainty.

1 Introduction

Consider the active system which consists of the management body-principal and the economic object - agent. Both, the principal and the agent have their own interests, which are reflected by their goal functions. In this game - theoretical framework the incentive problem is usually formulated as following: find feasible control variables (incentive function), which will induce the agent to undertake actions, which are the most preferable from principal's point of view [1,2].

We distinguish deterministic incentive problems (when all essential information about all the players, their goal functions, feasible sets and about the environment is common knowledge) and incentive problems under uncertainty [2]. In the latter case uncertainty may arise due to the lack of information about the state of nature (theory of contracts [3-5,10]), asymmetric information about players preferences (implementation theory [7]) and so on. As there exist hundreds of papers on incentives under interval and stochastic uncertainty, the more so it seems rather surprising that fuzzy incentive problems have not so far attracted enough attention of scientists (exception is the attempt to generalize classical theory of teams [6] to the case of the fuzzy environment [8]). Therefore this paper is devoted to the exploration of incentives under fuzzy uncertainty. Solution of the incentive problem implies the search of the optimal incentive scheme and investigation of its properties: coordinatability, nonmanipulability, dependence on information, uncertainty, etc.

2 Deterministic incentive problem

Consider the active system, which consists of the principle and one agent. Agent's strategy is the choice of action $y \in A$, principal's strategy is the choice of incentive function (penalty function) $\chi(y) \in M = \{ \chi(y): 0 \le \chi(y) \le C\}$.

When the action y is chosen (which in the deterministic model coincides with the output $z \in A_0$, $A_0 = A$), the agent's income is $h(y)$, while principal's income is $H(y)$. Thus goal functions are:

$$F(y) = H(y), \tag{1}$$

$$f(y) = h(y) - c(y) \tag{2}$$

It is worth noting that in the theory of contracts the penalty function is usually added to principal's income. Let $h(\cdot)$ be quasi-singlepeaked function [2] with a finite peak $y' = \arg\max_{y \in A} h(y)$ and $h(y') < +\infty$. Define the set of the implementable actions $P(c)$ [2]:

$$P(c) = \operatorname{Arg}\max_{y \in A} f(y) \tag{3}$$

and denote $P = \bigcup_{c \in M} P(c)$. Then the efficiency (guaranteed efficiency) of management is given by:

$$K_0 = \max_{c \in M} \max_{y \in P(c)} H(y);$$

$$K^g_0 = \max_{c \in M} \min_{y \in P(c)} H(y). \tag{4}$$

Maximum over the set of implementable actions is adequate if the hypothesis of agent's benevolence (HB) towards the principal is valid [1,2]. Let

$$x^- = \min \{x \in X: h(x) \ge h(y') - C \},$$

$$x^+ = \max \{ x \in X: h(x) \ge h(y') - C \},$$

$$c_c(x,y) = \begin{cases} C, & y < x \\ 0, & y \le x \end{cases} \tag{5}$$

$$c_k(x,y) = \begin{cases} C - h(y') + h(y), & y \in [x^-,x^+] \\ 0, & y \notin [x^-,x^+] \end{cases} \tag{6}$$

The jump penalty function (5) (C-type) and compensative penalty function (6) (K-type) are optimal in the considered model.

Theorem 1. i) $\forall\ c \in M\ \ P(c) \subseteq P = [x^-, x^+]$;

ii) $P(c_c) = \bigcup_{x \in A} P(c_c(x,x)) = P(c_k) = [x^-, x^+]$;

iii) $K_0 = \max_{x \in P} H(x)$.

It follows from the theorem 1, that C- and K-type penalty functions are characterized by the largest sets of implementable actions. Moreover, other optimal incentive schemes lie "between" them [2].

3 Fuzzy incentive problem

Below we describe the generalization of the deterministic incentive problem to the case of the fuzzy uncertainty about internal an external parameters of the system. The information is assumed to be symmetric.

3.1 Internal fuzzy uncertainty

Let agent's income function be fuzzy: $\underline{h}: A \times \Re^1 \rightarrow [0,1]$. If the penalties are unfuzzy, then according to the generalization principle [11] agents goal function $\underline{f}$ is given by

$$\underline{h}(y, u + c(y)). \tag{7}$$

Two feasible actions are compared by the following fuzzy preference relation (FPR) [9]:

$$\mu_{\underline{f}}(y_1,y_2) = \sup_{t \geq z} \min\{\underline{h}(y_1, t + c(y_1)), \underline{h}(y_2, z + c(y_2))\}. \tag{8}$$

The degree of certain alternative's $x \in A$ undominance is determined as

$$\mu_{\underline{f}}^{UD}(x) = 1 - \sup_{y \in A} [\sup_{t \geq z} \min\{\underline{h}(y, t + c(y)), \underline{h}(x, z + c(x))\} - \sup_{z \geq t} \min\{\underline{h}(x, z + c(x)), \underline{h}(y, t + c(y))\}]. \tag{9}$$

Rational choice of the agent is assumed to belong to the set of the maximally undominated alternatives [11]. Let $\forall\ y \in A\ \underline{h}(y,u)$ are normal functions [9].

Lemma 2. If the pair (y_0, u_0) is a solution of the following unfuzzy mathematical programming problem:

$$\begin{cases} u \rightarrow \max \\ \underline{h}(y, u + c_c(x,y)) = 1, \\ u \in \Re^1 \end{cases} \tag{10}$$

then y_0 is unfuzzy undominated action.

It is obvious that (10) covers all the unfuzzy undominated actions as if $\underline{h}$ is 1-normal then there are no unfuzzy undominated actions, which satisfy (10). Lemma 2 allows to define constructively the choice of the agent with the given penalty function. Note, that decreasing the uncertainty (going over to the deterministic model) the set of unfuzzy undominated alternatives coincides with the set of implementable actions.

Let $h(y)$ be some unfuzzy income function. Fuzzy income function $\underline{h}(y,u)$ is coordinated with h(y), iff:

$\forall$ y $\in$ A

1) $\underline{h}(y, h(y)) = 1$;

2) $\forall\ u_1, u_2:\ u_1 \leq u_2 \leq h(y)\ \ \underline{h}(y,u_1) \leq \underline{h}(y,u_2)$;

3) $\forall\ u_1, u_2: h(y) \leq u_1 \leq u_2\ \ \underline{h}(y,u_1) \geq \underline{h}(y,u_2)$.

Fuzzy income function which is coordinated with the unfuzzy quasi-singlepeaked income function is also referred to as quasi- singlepeaked.

Theorem 3. If the fuzzy income function is quasi-singlepeaked, then C-type incentive function is optimal.

Define $Q(x) = \{ y \in A : (y, f(x,y))$ *- satisfies (10)* $\}$, $x \in A$. The set of implementable actions is $Q = \bigcup_{x \in P} Q(x)$.

Theorem 4. i) Guaranteed efficiency in the active system with quasi-singlepeaked fuzzy income function does not exceed the deterministic one.

ii) If the HB is valid then the efficiency in the active system with quasi-singlepeaked fuzzy income function is greater then deterministic one.

Corollary 5. If $\underline{h}(y,u)$ satisfies: $\forall$ y $\in$ A $\underline{h}(y,u)=1$ iff $u=h(y)$ and $\underline{h}(y,u)$ is coordinated with this quasi-singlepeaked income function then the efficiency in the active system with quasi-singlepeaked fuzzy income function equals the deterministic one.

Let us introduce the criterion for the comparison of fuzzy uncertainties in two active systems which differ only by fuzzy income functions of the agents. Assume that $\underline{h_1}(y,u)$ and $\underline{h_2}(y,u)$ are two fuzzy income functions, coordinated with the same unfuzzy quasi-singlepeaked income function $h(y)$. The first active system is characterized by lower uncertainty if

$$\forall\ y \in A,\ \forall\ u \in \Re^1\ h_1(y,u) \leq \underline{h_2}(y,u). \tag{11}$$

Theorem 6. i) Guaranteed efficiency does not increase with the growth of uncertainty in the active system with quasi-singlepeaked fuzzy income function.

ii) If the HB is valid then the efficiency increases with the growth of uncertainty in the active quasi-singlepeaked fuzzy income function .

3.2. External fuzzy uncertainty

Consider the model with external uncertainty, where agent's action jointly with the state of nature $\theta \in \Omega$ determines the output $z \in A_0$, $z = z(y, \theta)$ (compare this model with the approaches of the theory of contracts [3-5]). Principal's goal function $F(y)$ depends on the agent's actions, while agent's goal function

$$f(z) = h(z) - c(z) \tag{12}$$

depends on the output, which is observed by the principal (agent's action is unobservable to the principal). Suppose that the principal and the agent has the same fuzzy information about the state of nature: $\underline{P}: A_0 \times A \rightarrow [0,1]$. Agent's goal function (12) and information function $\underline{P}$ induce on the set of feasible actions the following FPR $\underline{R}$ [9]:

$$\mu_{\underline{R}}(y_1, y_2) = \sup_{z,x \in A} \min[\underline{P}(x,y_1), \underline{P}(z,y_2), \mu_f(x,z)]. \tag{13}$$

The degree of $x \in A$ undominance equals

$$\mu_{\underline{R}}^{UD}(x) = 1 - \sup_{y \in A} [\sup_{z,t \in A:\ f(z) \geq f(t)} \min\{\underline{P}(z,y), \underline{P}(t,x)\} -$$

$- \ sup_{z,t\,Î A:\ f(t)\ ³\ f(z)}\ min\{\underline{P}(z,y), \underline{P}(t,x)\}$

Rational choice of the agent belongs to the set of maximally undominated actions A^{UD} (which maximize (14)). As the set of undominated actions depends on the incentive scheme, then $P(c) = A^{UD}(c)$.

Assume that $A = A_0$ is a closed interval in $Â^1$; fuzzy sets $\underline{P}$(z,y) are 1-normal (i.e. *" y Î A* $sup_{z\,Î A}$ *$\underline{P}$(z,y)=1 and " z Î A $ y Î A: $\underline{P}$(z,y) = 1)* and P is upper semicontinous.

Application of (14) for the calculation of the undominance degree is rather complex. The result of the following lemma simplifies the analysis.

Lemma 7. If (z_0, y_0) is a solution of the following unfuzzy mathematical programming problem:

$$\begin{cases} f(z)\ ®\ max, \\ \underline{P}(z,y)=1, \\ y\,Î A,\ z\,Î A, \end{cases}$$

then $m_{\underline{R}}^{UD}(y_0) = 1$, i.e. y_0 - unfuzzy undominated action.

In accordance with the introduced assumptions the set of unfuzzy undominated actions is not empty. If C-type penalty function is used then maximum of (12) may be achieved only on the interval $[x^-, x^+]$ (see Th.1). Choose some x∈ $[x^-, x^+]$ and define

$Q(x) = \{ y\ Î\ A: \underline{P}(x,y) = 1 \}$.

Lemma 8. If the HB is valid then for any $x\ Î\ P$ and for any $y\ Î\ Q(x)$ there exists incentive scheme $\chi \in M$ (namely, χ_c), such that the action y belongs to the set unfuzzy undominated actions.

Lemma 9. If the HB is valid then

$$S = È_{c\,Î M}\, P(c) = È_{x\,Î P}\ Q(x);$$

if HB is not valid then

$$S = È_{x\,Î P}\ min\{ y\,Î\ A:\ y\,Î\ Q(x) \}.$$

Theorem 10. The solution of the incentive problem in the active system with external fuzzy uncertainty coincides with the solution of the following problem:

$$F(x)®\ max_{x\,Î S}$$

Corollary 11. For any incentive scheme $c\ Î\ M$ there exists C-type incentive scheme of the same efficiency.

Thus, when solving the incentive problem in the active system with external fuzzy uncertainty, one can restrict his attention to jump penalty functions. Denote K and K^g - efficiency and guaranteed efficiency of management.

Theorem 12. $K^g\ £\ K_0^g,\ K\ ³\ K_0$.

Consider two active systems which differs only in fuzzy information functions $\underline{P}_1(z,y)$ and $\underline{P}_2(z,y)$. The uncertainty is lower in the first system if ∀ y ∈ A, ∀ z ∈ A_0 $\underline{P}_1(z,y) \le \underline{P}_2(z,y)$. Denote K_1 and K_2 corresponding efficiencies.

Theorem 13. $K_1^g\ ³\ K_2^g,\ K_1\ £\ K_2$.

Results of theorems 12 and 13 confirm the intuitive understanding of the uncertainty role: the efficiency (guaranteed) of management under uncertainty does not exceed deterministic one and decreases with the growth of uncertainty. This result is also valid for the guaranteed efficiency of management in most of the active systems, which operate under intreval and stochastic uncertainty [2,9].

4 Conclusion

Thus we proved that C-type incentive functions, which are optimal in deterministic case [1,2], are also optimal in most systems with fuzzy uncertainty. The result on the uncertainty influence on the management efficiency (the guaranteed efficiency of management under fuzzy uncertainty is lower than the deterministic one and decreases with the growth of uncertainty) corresponds to the common sense [9]. Moreover, exploration of the problems lead to some nontrivial results. For example, implying the HB to be valid, the management efficiency increases with the growth of the uncertainty.

We hope that further development and deeper exploration of incentives in fuzzy active systems will allow to create a unified incentive theory, which will embrace the active systems theory, the theory of contracts, the implementation theory, etc.

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