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# HIERARCHICAL MODELS IN MODERN CONTROL THEORY

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*Annotation*: Historical and modern approaches towards heterogeneous models and hierarchical modeling in control theory are listed. A unified approach towards a design of hierarchical models of multiagent systems is described and illustrated by the example of diffuse bomb model. Problems of "hierarchical automation" in organization-technical and production systems are described. General problems of hierarchical modeling are underlined.

*Keywords*: heterogeneous model, hierarchical modeling, multiagent system, diffuse bomb problem, production systems.

## 1. Introduction: Heterogeneous Models and Hierarchical Modeling

In recent years, control theory more and more addresses the term of system "*heterogeneity*" comprehended, in the first place, as the multiplicity of its mathematical description (e.g., descriptive dissimilarity of separate subsystems: the type and scale of time/space of subsystems functioning, multi-type descriptive languages for certain regularities of a studied object, etc.) [8]. "Heterogeneity" also means complexity appearing in (qualitative, temporal and functional) *dissimilarity*, (spatial and temporal) *distribution* and the *hierarchical*/*networked structure* of a controlled object and an associated control system.

An adequate technology for design and joint analysis of a certain set of heterogeneous systems models is the so-called *hierarchical modeling*. According to this technology, models describing different parts of a studied system or its different properties (perhaps, with different levels of detail) are ordered on the basis of some logic, thereby forming a hierarchy or a sequence (a horizontal chain). Generally, lower hierarchical levels correspond to higher levels of detail in modeled systems description. Each element of a sequence possesses almost same level of detail, and the results (outputs) of a current model represent input data for a next model. Such approach to modeling was born and further developed in the 1960–1970's, e.g. see [6].

In some sense, hierarchical models are a wider category than hybrid models and the multi-model approach. A *hybrid model* is a model combining elements of two or more models reflecting different aspects of a studied phenomenon or process and/or employing different apparatuses (languages) of modeling–see Fig. 1. For instance, a hybrid model can include discrete and continuous submodels, digital and analog submodels, and so on.

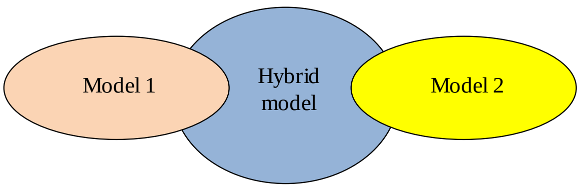


*Fig. 1. The narrow interpretation of a hybrid model*

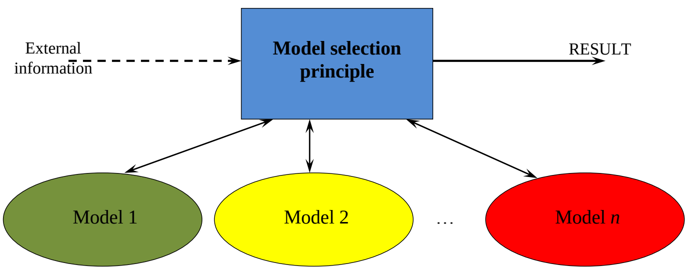


*Fig. 2. The modern interpretation of a hybrid model. The multi-model approach*

In the wider interpretation, a hybrid model represents a complex of models each chosen under well-defined conditions, see Fig. 2. As an example, consider hybrid dynamic systems (HDS, also known as switching systems). The expression in the right-hand side of the HDS differential equation is chosen from a given set of options depending on the current state of the system and/or time and/or auxiliary conditions.

Within the *multi-model approach*, several models are used sequentially or simultaneously with further or current analysis and selection of “best” results.

Hierarchical (sequential) models may have a more complex structure, see Fig. 3. At each level, a model can be hybrid or follow the multi-model approach. Hierarchical models lead to the problems of *aggregation* and *decomposition* well-known in mathematical modeling.

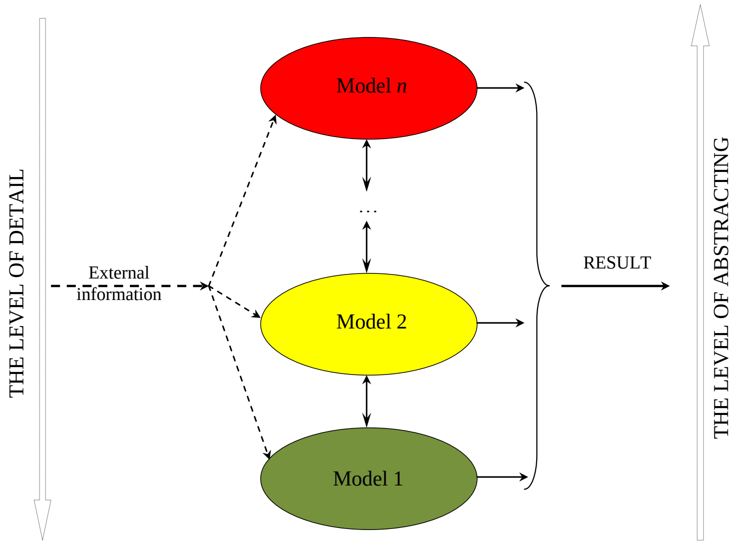


*Fig. 3. A hierarchical (sequential) model*

The next subsections give several examples of hierarchical models and the discussion of their drawbacks and advantages.

## 2. Hierarchical Models of Multiagent Systems

For the recent 15-20 years, a modern tendency in control theory has been seeking towards “*miniaturization*,” “*decentralization*” and “*intellectualization*” in systems of very many interacting autonomous *agents* having social, technical or informational nature. Inherent properties of multi-agent systems (MAS) such as *decentralized* interaction and agents’ multiplicity induce fundamentally new emergent properties (*autonomy*, smaller *vulnerability* to unfavorable factors, etc.) crucial in several applications [11, 12, 13].

In MAS the hierarchy of models is *inter alia* generated by the functional structure of the agent. The latter may have several hierarchical levels, see Fig. 4 [9]. The lowest (operational) level is responsible for implementation of actions (e.g., motion stabilization with respect to a preset path). Tactical level corresponds to actions’ choice (including interaction with other agents). Strategic level is in charge of *decision-making*, *learning* and *adaptivity of behavior*. And finally, the highest level (goal-setting) answers the principles of goal-setting and choice of the mechanisms of functioning for agents. The diffuse bomb model in section 3 implements the general structure described by Fig. 4.

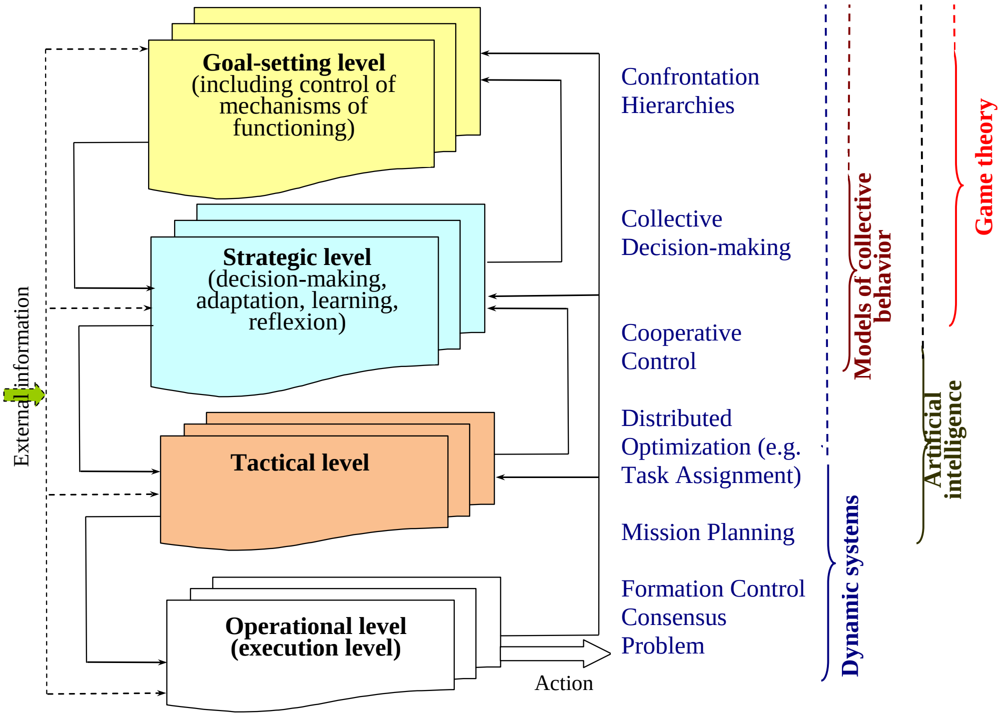


*Fig. 4. The hierarchical structure of an agent in MAS*

The structure presented by Fig. 4 seems rather universal. However, most realizations of multi-agent systems involve merely two lower levels and the framework of *dynamic systems theory*.

In *mission planning* problems, one can use different means of *artificial intelligence*, e.g., neural networks, evolutionary and logical methods, etc.

Also, let us mention *distributed optimization* (*agent-based computing*, see [2]) as a direction of modern optimization widespread in MAS. Its key idea consists in the following. An optimization problem of a multivariable function is decomposed into several subproblems solved by separate agents under limited information. For instance, each agent is "responsible" for a certain variable; at a current step, it chooses the value of this variable, being aware of the previous choice of some its "neighbors" and seeking to maximize its own local "goal function." Given an initial (global) goal function, is it possible to find the "goal functions" of agents and their interaction rules so that the autonomous behavior of agents implements a centralized optimum? (in *algorithmic/computational game theory* [1, 5], this optimum can correspond to a Nash equilibrium or a Pareto efficient state of agents' game).

Consider the strategic level of agent's architecture, which answers for adaptation, learning, reflexion and other aspects of strategic decision-making. *Game theory* and *theory of collective behavior* analyze interaction models for rational agents. In game theory, a common scheme consists in (1) describing the "model of a game," (2) choosing an equilibrium concept defining the stable outcome of the game and (3) stating a certain *control problem*–find the values of controlled "game parameters" implementing a required equilibrium [10].

Nowadays investigators gradually shift their efforts towards higher levels of agents' architecture, i.e., from consensus and communications problems to cooperative control and strategic behavior models of agents [3]. To justify this statement, let's analyze the networked control topics on main worldwide conferences on control theory and applications. Being subjective and not pretending to a complete overview, the author emphasizes triennial world congresses conducted by *International Federation of Automatic Control* (IFAC) and annual *Conferences on Decisions and Control* (CDC) under the auspices of *Institute of Electrical and Electronics Engineers* (IEEE). Alongside with these major events (or even jointly with CDC), there exist regular "national" conferences (actually, these conferences gather researchers from many other countries): *American Control Conference* (ACC) and *European Control Conference* (ECC). In the USSR, the role of such national conferences belonged to *AMCP* - All-Union Meetings on Regulation Theory (later, on Automatic Control and, then, on Control Problems).

Fig. 5 and Fig. 6 specify the topics of networked control by the levels of agents architecture in MAS and problems treated at these levels. The following groups of topics have been identified via expertise: MAS and *consensus* problems, communications in MAS, *cooperative control*, upper levels of control (*strategic behavior* of agents), “others” (mostly, information and communication networks with a slight emphasis on control problems).

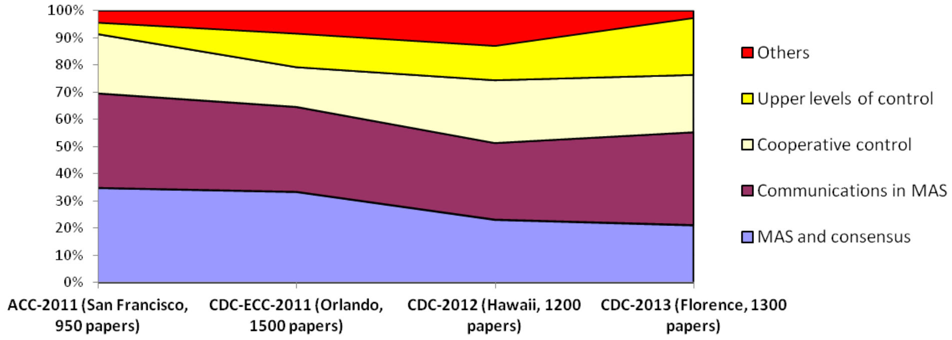


*Fig 5. Specification of networked control topics at ACC and CDC*

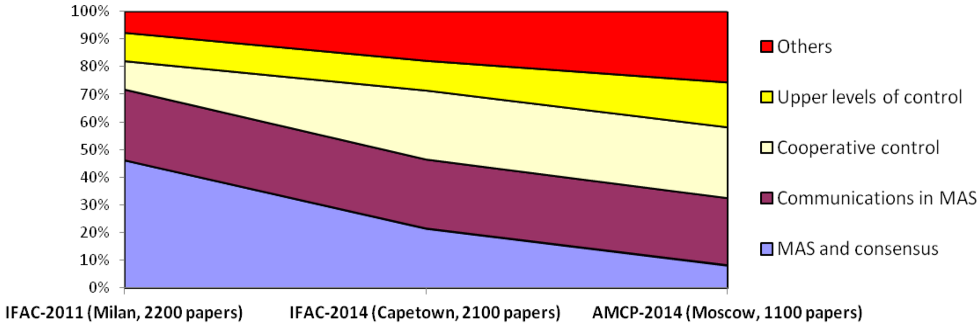


*Fig. 6. Specification of networked control topics at IFAC Congresses and AMCP-2014*

## 3. The Diffuse Bomb Model

An example of the hierarchical model of a MAS is the model of distributed penetration through a defense system (the so called diffuse bomb model [4]).

Consider a group of autonomous moving agents must hit a target with given coordinates. At each time step, any agent can be detected and destroyed by a defense system (with a certain probability). Detection/annihilation probability depends on agent’s coordinates and speed, as well as on the relative arrangement of all objects in the group.

The problem is synthesizing algorithms of decentralized interaction among agents and their decision-making (the choice of direction and speed of their motion) to maximize the number of agents reaching the target. Agents appear “intelligent” in the following sense. Some agents (reconnaissance) can acquire on-line information on the parameters of the defense system. By observing the behavior of the reconnaissance agents, the rest ones perform “reflexion,” assess the limits of dangerous areas and solve the posed problem.

The following hierarchical model defined by Table 1 serves for appraising and choosing most efficient algorithms of behavior in [4].

*Table 1. The diffuse bomb model*

| Hierarchical level | Modeled phenomena/processes | Modeling tools |
|---|---|---|
| 6 | Choosing the set of agents and their properties | Discrete optimization methods |
| 5 | Choosing the paths and speeds of agents | Optimal control |
| 4 | Agent’s forecast of the behavior of other agents | Reflexive games. The reflexive partitions method |

| Hierarchical level | Modeled phenomena/processes | Modeling tools |
|---|---|---|
| 3 | Detection probability minimization based on current information | Algorithms of course choice |
| 2 | Collisions avoidance, obstacles avoidance | Algorithms of local paths choice |
| 1 | Object's movement towards a target | Dynamic motion equations |

### 4. "Hierarchical Automation" in Organization-Technical Systems

Since the 1980's, *production systems* have followed a long path from flexible to holonic systems. In recent years, they attract the growing interest of researchers in connection with new market challenges: the efficiency of production specialization and decentralization, product and service differentiation, etc. There appear *networked productions* and "*cloud*" *productions*. Along with implementation of fundamentally new technologies of production (nanotechnologies, additive technologies, digital production, and so on), we observe gradual changes in its organization, i.e., the emphasis is shifted from operations automation to *control automation* at all life cycle stages.

Existing challenges such as:

– a huge number of product's customized configurations;
– integration of small- and large-scale production;
– lead-time reduction for an individual order;
– supply chains integration for stock optimization;

and others call for solutions guaranteeing:

– the universality of production systems and their separate components;
– the capability of rapid and flexible adjustment with respect to new tasks;
– autonomous decision-making in production owing to high-level control automation;
– survivability, replicability and scalability owing to network-centric control and multi-agent technologies;
– decision-making in production with proper consideration of economic factors, etc.

Modern production systems have a hierarchical structure, as indicated by Fig. 7. And the complexity of control problems treated induces their decomposition into decision-making levels. Each level in control problems solution corresponds to its own goals, *models* and *tools* (Fig. 7) at each stage of control (organizing, planning, implementing, controlling and analyzing). Hence, in organizational-technical production systems it is possible (and necessary) to apply hierarchical modeling.

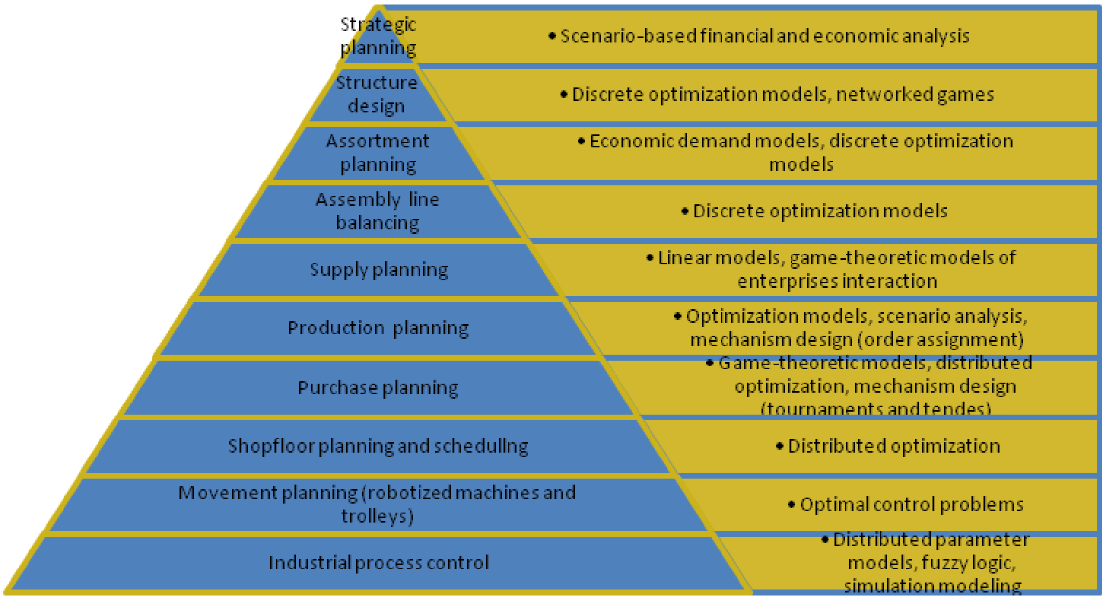


*Fig. 7. Hierarchical models in production systems*

This possibility is implemented, but on an irregular and unsystematized basis. Obviously, one can solve real problems of automation, analysis and decision support for production systems only within appropriate computer-aided informational systems. As an illustration, consider the classes of such systems in the ascending order of their "hierarchical level":

– lower-level control systems (PLC, MicroPC, …);

– supervising and scheduling systems (SCADA, DCS, …);

– production planning and management systems (MRP, CRP, …, MRP2, …);

– integrated systems (MES, …, ERP., …);

– systems responsible for interaction with an external environment or development (SCM, CRM, PMS, …);

– upper-level analytic systems (OLAP, BSC, DSS, …).

These classes of systems use mathematical models, but very sparsely; as a rule, the higher is the level of hierarchy (this statement is true for separate informational systems and for integrated informational systems of product life cycle management (PLM) including computer-aided design systems, which realize the complex of the listed functions), the lesser is their usage. For instance, lower-level controllers employ in full automatic control theory; project management systems (PMS) incorporate classical algorithms for critical path search, Monte Carlo methods for project duration estimation, and heuristics for resources balancing; ERP systems and logistics systems (SCM) involve elementary results from stock management theory, and so on.

Nevertheless, full-fledged implementation of the so-called "*hard*" *models* and "quantitative science" (operations research, discrete optimization, data analysis and other branches of modern applied mathematics) in informational systems still waits in the wings.

Several global problems exist here. On the one hand, mathematical models require very accurate and actual information often associated with inadmissibly high organizational and other costs. On the other hand, in many cases "*soft*" *models* (putting things in order in production processes, implementation of typical solutions and standards in the form of qualitative best practices, etc.) yield an effect exceeding manyfold the outcomes of quantitative models, yet consume reasonable efforts. Therefore, it seems that quantitative models should be applied at the second stage, "extracting" the remainder of potential efficiency increase.

## 5. Conclusion

The forthcoming years will be remarkable for transition in control theory from the so-called *$C^3$ paradigm* (joint solution of *Control* + *Computations* + *Communications* problems) to *the $C^5$ concept* (*Control* + *Computations* + *Communications* + *Costs* + *Life Cycle*), when the above-mentioned problems are solved taking into account cost aspects (in the general sense) over the whole *life cycle* of a system including the joint design of a control system and its controlled object.

Speaking about "*networkism*," we have to touch "*network-centrism*" (network-centrism operates its own abbreviations differing from control theory (see above $C^3$ or $C^5$): $C^3$I–Command, Control, Communications and Intelligence, $C^4$I–Command, Control, Communications, Computers and Intelligence, and others) extremely fashionable nowadays (also called "*network-centric fever*"). It admits several interpretations covering organization and analysis principles of any networks in principle or temporary networks created for specific task or mission execution at a right place and right time (*networked organizations*, e.g., interaction of military units in a combat theater). This approach finds wide application in network-centric warfare problems for vertical and horizontal integration of all elements during a military operation (control, communication, reconnaissance and annihilation systems).

Another manifestation of "*networkism*" concerns the growing popularity of *distributed decision support systems*. The intensive development of information and communications technologies (ICT) increases the role of informational aspects of control in decentralized hierarchical systems (an example is decision-making support in distributed decision systems which integrate heterogeneous information on strategic planning and forecasting from different government authorities and industrial sectors). One of such aspects consists in *informational control* as a purposeful impact on the awareness of controlled subjects; therefore, a topical problem is to develop a mathematical apparatus providing an adequate description for an existing relationship between the behavior of system participants and their mutual awareness [10].

Design of intelligent analytic systems for informational and analytic support of goal-setting and control cycle represents another important informational aspect of control in decentralized hierarchical systems. Here it seems necessary to substantiate methodological approaches to control efficiency in decentralized control systems, including elaboration of principles and intelligent technologies for data acquisition, representation, storage and exchange.

We underline that an appreciable share of information required for situation assessment, goal-setting and control strategy choice in decentralized systems is ill-structured (mostly, in the form of text). And there arise the problems of relevant search and further analysis of such information [7]. The described circumstances bring to the need for suggesting new information retrieval methods (or even knowledge processing methods) based on proper consideration of its lexis and different quantitative characteristics and, moreover, on analysis of its semantics, separation of target data and situation parameters, assessment of their dynamics and scenario modeling of situation development in future periods.

Concluding this paper dedicated to heterogeneous models and hierarchical modeling, we underline a series of their common classes of problems. Modern controlled objects are complicated so that sometimes a researcher would hardly separate out purely hierarchical or purely networked components. In such cases, it is necessary to consider *networks of hierarchies* and *hierarchies of networks* [8].

First, at each level models have their own intricacies induced by a corresponding mathematical apparatus. Moreover, there arise “conceptual coupling” dilemmas and the *common language* problem among the representatives of different application domains.

Second, a complex of “*joined*” *models* inherits all negative properties of each component. Just imagine that, at least, one model in a “chain” admits no analytic treatment; then the whole chain is doomed to simulation modeling. The speed of computations in a chain is determined by the slowest component, and so on.

And third, it is necessary to assess the comparative efficiency of the solutions of aggregated problems, as well as to elaborate and disseminate typical solutions of corresponding control problems in order to transfer them to the engineering ground.

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