

Models of Optimal Organizational Hierarchies

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We combine the transaction costs approach with the original mathematical results in an optimal hierarchy design (Mishin S.P., 2004; Goubko M.V., 2006) to formulate and investigate the models of multi-layer management hierarchies. Our approach assumes reducing the problem of rational management hierarchy to the model of discrete optimization (to choose a hierarchy from a finite set of feasible hierarchies with the aim to maximize a certain criterion that maps the set of feasible hierarchies into the number scale).

Formal models of intra-firm hierarchies are studied by economists since the 60s of XX century. The main issue addressed by the earlier papers (see M. Beckmann (1960), O.E. Williamson (1967), G.A. Calvo and S. Wellisz (1978)) is whether the loss of control in a management hierarchy limits the growth of a firm. The later literature pays more attention to the explanation of empirical data on executives' compensation in big firms (S. Rosen (1982), V. Smeets and F. Warzynski (2006)). These issues are important for the theory of the firm but business administration still seeks for formal normative models offering a clue to the formation of competitive management hierarchy.

We propose a normative model of management hierarchy optimization. The questions posed by this model are: how many managers the firm must hire, when headcount should be increased or decreased, how managers wages depend on their positions, whether corporate information systems implementation results in a flatter management hierarchy, when the growth of the firm is advantageous, etc.

We study a manufacturing firm that chooses one of available products to produce. The product determines the technology and, thus, the set of productive workers. The hierarchy of managers built over the set of workers provides monitoring and coordination. Two central points of management hierarchy are considered – the maintenance costs and the loss of control in a chain of command.

The maintenance costs of a manager depend on her span of control, position (size of a unit under control), monitoring efforts, and the firm's production plan. The costs also depend on two external parameters – the level of managers' ability and the degree of standardization of business processes in the firm. The wastes from the loss of control increase exponentially with a hierarchy level.

The problem in hand is the problem of the principal – to choose a product, production plan, and to organize the efficient execution of this plan, i.e. to find out how many managers to hire, how to subordinate both workers to managers and managers to higher-layer managers in order to obtain better efforts (and thus, the output) at lower costs.

We consider a framework with the manager's cost function obeying constant elasticity with respect to the size of a unit under control. For such setting the technique has been developed earlier (Goubko, 2006) to solve the optimal hierarchy problem. We prove the optimal management hierarchy to be uniform (i.e. every manager in a hierarchy has the same span of control) and symmetric (i.e. every manager seeks to divide the subordinate group of workers equally among his immediate subordinates). Then the formulas are obtained for the optimal span of control, the managers' headcount, their compensation amounts, and a production plan. Thus, for the current setting the problem of optimal hierarchy analysis and synthesis is completely solved.

The results allow analyzing the impact of external parameters (such as managers' ability and the degree of standardization) on a firm's size, its financial results, employees' wages and the shape of the optimal hierarchy. Prospective studies may be devoted to the model identification from empirical data and business administration literature.

The mathematical technique developed can be used to solve optimal hierarchy problems in various areas: design of data collection and quality management systems, assembly planning, etc.

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